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TECHNICAL REPORT 4945

MOMENTS-II

A COMPUTER PROGRAM TO CALCULATE

MOMENTS AND PRODUCTS OF INERTIA OF

ASYMMETRIC OBJECTS

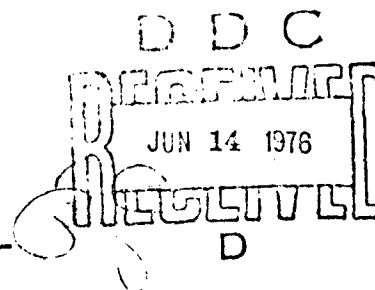EDWARD B. LACHER

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This report describes the computer program MOMENTS-II, which is a greatly modified and extended version of the computer program MOMENTS, described in Picatinny Arsenal Technical Report 4143, July 1971. The present report supersedes the previous publication. MOMENTS-II is capable of calculating the components of the inertia tensor (moments and products of inertia) of combinations of arbitrarily located and oriented asymmetric objects with respect to ─

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20.   ABSTRACT

arbitrary rectangular systems; as well as of calculating other
physical properties such as mass, center of gravity and principal
axes of these objects. The report also describes the model
employed and the analysis leading to the derivations of the formulae
used by MOMENTS-II, and for convenience is divided into three
chapters containing information specifically directed towards the
program user, programmer, and analyst respectively.

PREFACE

This report is a complete self-contained document which describes all aspects of the method and associated program MOMENTS-II for computing various properties of asymmetric objects. It is divided into three chapters, entitled User's Guide, Programmer's Guide and Analyst's Guide, which as the titles indicate, are directed towards the user, programmer and analyst respectively. The first chapter explains the basics of the program and should therefore be read by those in all three of these categories; this chapter alone should suffice to enable one to use the program successfully. The programmer, who may wish to modify the coding, should read Chapter 2 also, and the analyst, who may wish to examine the derivations of the formulae used by the program, should read Chapter 3 also.

The numbering system is the standard type, where the designation 1.2.3 refers to sub-section 3 of section 2 in Chapter 1. The equations are all in Chapter 3 and they are numbered consecutively from (1) to (74).

In general, any notation relating especially to the content of a single section of the report is summarized at the beginning of that section. Also, certain terms such as the word "Part", have been written with an initial capital letter to indicate that they have special meanings in this report. Finally, all FORTRAN names and variables have been written completely capitalized as per standard practice for easy identification.

The Appendices contain the listings of the input cards and the output from each of the four sample cases explained in 1.5.1 to 1.5.4.

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INTRODUCTION

The purpose of the computer program MOMENTS-II is to calculate certain physical and geometric properties, such as moments and products of inertia, center of gravity, mass, volume and principal axes of asymmetric bodies. It differs from other programs in that MOMENTS-II actually handles a wide range of asymmetric objects and it can calculate the moments and products with respect to arbitrary rectangular coordinate systems.

The present program is an expanded and greatly modified extension of MOMENTS as described in Ref. 2. The capabilities of the present version, beyond those of the original consist of accepting more types of solids and of rotating and/or translating moments and products to arbitrary coordinate systems, each described by their origin and direction cosines. The inclusion of new types of solids has been directed towards applying the program to analyzing objects, working from their blueprints. The rotating and translating capability greatly increases the flexibility and utility of the program, for it allows one to describe each Part (for input purposes) with respect to convenient sets of axes, rather than requiring that all Parts be described with respect to some one fixed system. The simple computation of the three Euler angles which was contained in the original version has been removed because of the great variety of definitions of these angles used in practice. It is more reasonable that the user who requires the calculation of these angles add his own subroutine to compute them according to the definition he prefers.

The method used by MOMENTS-II requires that the object to be analyzed be represented or modelled by combinations of certain types of solids, which can be located and oriented in an arbitrary manner. The great versatility of this program is derived from such considerations as the following: the allowable solids include both individual and combinations of different sectors of frusta of cones which are not symmetric about any axis, rather than restricting the program to complete frusta, and the allowable solids include both wedges and trapezoidal prisms, rather than being restricted to rectangular parallelepipeds. The present program is in this respect significantly more general than other programs of its kind which can generally handle composite objects which must be represented as collections of simple axi-symmetric solids only.

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The major present applications of MOMENTS-II are in measuring shell imbalances for determining proper machine tolerances for shell dimensions, and in computing the moments and products of inertia of objects (such as rotors) which are used in fuze mechanisms. However, a wide range of applications are possible, and any field requiring the calculation of the properties computed by MOMENTS-II could make use of the program.

1.0 USER'S GUIDE

This chapter provides the information necessary to
use the computer program MOMENTS-II to calculate the
moments and products of inertia and certain other
physical properties of solid objects, with respect to
any rectangular coordinate system. It explains the basic
model used by the program, describes the "Parts" into
which all objects to be analyzed must be modelled, and
contains an input guide, output guide and (explained)
sample cases. This chapter should be considered
as a single unit, in that no portion of it short of the
complete chapter should be expected to suffice as an
explanation of how one uses MOMENTS-II.

1.1 MOMENTS-II MODEL

This section describes the model employed by
MOMENTS-II in calculating the various properties of
objects. Sub-section 1.1.1 contains a list and a
brief definition of the various properties which
MOMENTS-II is capable of calculating as well as the
notation used in Chapter 1 to represent them. In
order to compute these properties for a given object,
that object must first be "modelled" into (i.e., decom-
posed into or approximated by) any number of specific
types of solid bodies which shall be called "Parts" and
which may be considered as the building blocks upon which
the MOMENTS-II program is based. Sub-section 1.1.2
contains an explanation of this concept and how Parts
are described to the program, and 1.1.3 contains a
further explanation of the Rotation/Translation infor-
mation used to describe their position and orientation.
This requires the use of various coordinate systems,
and sub-section 1.1.4 briefly characterizes those systems
used by MOMENTS-II.

1.1.1 Notation for Physical Properties

The following table lists all the properties of objects with which MOMENTS-II is concerned. In general, the density p is given, whereas the other properties listed below must be computed (for each Part as well as for the entire body). Those quantities denoted by (*) are computed solely for the purpose of calculating the ones appearing on the line immediately following them.

Symbol	Explanation	Mathematical Definition
p	Density	
V	Volume	$\iiint dxdydz$
m	Mass	$\iiint p\,dxdydz$
Myz, Mxz, Mxy(*)	First moments of inertia about yz, xz and xy-planes	$\iiint x\,dm$, etc.
$\bar{x}, \bar{y}, \bar{z}$	Coordinates of center of gravity	Myz/m, etc.
Ix^2, Iy^2, Iz^2(*)	Second moments of inertia about yz, xz, and xy-planes	$\iiint x^2\,dm$, etc.
Ixx, Iyy, Izz	Moments of inertia about x, y, and z-axes	$\iiint (y^2+z^2)\,dm$, etc.
Ixz, Iyz, Ixy	Products of inertia	$-\iiint xz\,dm$, etc.

1.1.2 Method of Parts

One uses the MOMENTS-II program to compute certain properties of objects with respect to an arbitrary set of rectangular coordinate axes, called "Reference Axes", which are chosen by the user. Before the program can compute the properties of an object, one must first describe to it the size, shape and density of the object as well as the location and orientation of the object in the Reference System. In order to accomplish this, the user must represent or approximate the actual object by a combination of any number of the following four specific types of solids, namely sectors of frusta of cones (sometimes referred to as conical wedges) and combinations of sectors, right angle wedges (and truncated wedges), angular wedges, and "concave sectors". These four types of solids, generically called "Parts", will be referred to as "Basic Parts", "Standard Wedges", "Angular Wedges", and "Concave Parts" respectively. They will be described in detail in section 1.2 and are illustrated in Figures 2 to 5. Thus, in order to use MOMENTS-II to calculate the required properties (as listed in 1.1.1) for a particular object, that object must first be approximated by a combination of one or more of the four types of Parts listed above. By describing each of the Parts making up the object individually (i.e., shape, size, density as well as location and orientation in the Reference System) to the program, one is thereby providing it with a complete description of the model of the actual object. MOMENTS-II can then calculate the properties of these Parts one at a time and combine them so as to obtain the properties of the composite object. The resulting properties computed by MOMENTS-II are virtually exact for the model or combination of Parts described by the user, but the accuracy relative to the actual object is determined by the quality of the model the user has constructed to represent it. It will be seen that the Parts available for modelling objects are sufficiently flexible so that they can be used to quite accurately represent a wide class of objects.

From these remarks one can observe that the basic tasks of the user are the modelling of the object by Parts and the describing of the object to MOMENTS-II, the latter being equivalent to describing the Parts comprising the object to the program. The first task is illustrated in the sample cases in 1.5 and the second is the subject of the remainder of this sub-section.

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An object is considered to be completely described to MOMENTS-II when the type, dimensions, location and orientation (with respect to the Reference System) of each Part making up the object have been specified. The type and dimensions of a Part are specified by what shall be called Dimensional information. To simplify the method of describing the location and orientation of the Parts with respect to the Reference System, a slightly indirect approach is taken. Each Part is assigned a set of three orthogonal axes called "Input Axes", which, when chosen by the user subject to certain restrictions, are then rigidly fixed to the Part. One describes the position and orientation of the Part in the Reference System by specifying the position and orientation of the Part's attached Input System with respect to the Reference System. This data used to describe each Part may then be regarded as consisting of three basic types, Dimensional information, Input System information and Positional or "Rotation/Translation" information.

The Dimensional information consists of the values for the quantities which describe the shape, density and size of Parts. Examples are the specification of which type of solid the Part is, as well as such quantities as lengths, widths, densities, radii and heights.

The Input System information consists of all data describing the location and orientation of the Part with respect to its Input System, even though it may (implicitly) contain some information of a Dimensional nature also. Because the direction of a Part's Input System z-axis is fixed relative to the Part and because it is required that this axis pass through a particular point in the Part, the user must always specify the lower z-coordinate (DZ) of the Part (to locate it along the z-axis), and except for the Standard Wedge, also the two angles $TH(1)$ and $TH(2)$ which define the rotational position of the Part about this z-axis. These quantities DZ, $TH(1)$, and $TH(2)$ are referred to as Input System data even though when specifying the lower and upper angles one then is fixing not just the angular position of the Part (or sector of a Part) but also its angular measure $TH(2) - TH(1)$, which is actually one of its dimensions.

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Finally, the Positional or Rotation/Translation
information locates the Input System relative to the
(fixed) Reference System, by specifying the direction
cosines of the Input Axes with respect to the Reference
System (Rotation information) and/or the origin of the
Input System in Reference System coordinates (Translation
information). In short, a Part is then unambiguously
described and located in relation to the Reference
System by its Dimensional, Input System, and Positional
information.

It should be evident from the definition of Input
System information that the method of locating a specific
Part is not unique, but rather, since the user has some
freedom in positioning a Part in its Input System, there
are an infinite number of ways of describing any one Part.
The major reason for such a formulation is that it facili-
tates the inputting of objects with constant cross sections
(in the xy-plane) which can almost be considered as two-
dimensional objects. This is illustrated in Sample Case 3
(see 1.5.3) in which all Parts have their input Axes parallel
to the Reference Axes. In such a case the use of the
minimum z-coordinate DZ may allow one to describe Parts
making up an object with a minimal amount of Translation
information, because when the only displacement of the
Input System origin is in the z-direction (that is, when
the origin of the Part's Input System lies on the Reference
System z-axis), DZ alone can serve to describe this dis-
placement without requiring any Translation information.
More than half of the Parts in Sample Case 3 use DZ to make
Translation information unnecessary. Furthermore, the use
of the angles TH(1) and TH(2) allows one to describe all
of the Parts making up an object whose Input z-axis is
parallel to the Reference z-axis without the need for Rotation
information. In fact, none of the Parts in Sample Case 3
require Rotation information at all for this reason. Even
in cases where the above special conditions hold for either
the x or y-axis instead of for the z-axis, simple Rotation
information describing merely interchanges of axes will
suffice in place of more complicated Rotation information.
If one insists on removing this non-uniqueness, one can
choose the Input System in such a way as to require that
the lower angle TH(1) (where applicable) and the minimum
Input System z-coordinate DZ of any Part both be zero always.
This would remove any control the user has in choosing the
Input System and would insure that descriptions of inputted
Parts would be unique; but as stated above, it would have
the disadvantage of making Rotation/Translation information
necessary for almost every Part. This is equivalent to
requiring nine direction cosines instead of just one angle,
and three coordinates of a point instead of just one
displacement, in almost all cases.

Once MOMENTS-II has available to it the Dimensional and Input System information for a Part, the program first calculates its properties with respect to its Input Axes and secondly uses the Positional information (if included) to calculate these properties with respect to the Reference System. These calculations are repeated for each Part (except see below for "Known Parts") and thirdly the program sums these properties to obtain those of the entire object with respect to the Reference System. Finally, it calculates the center of gravity and the principal moments with respect to the CG-System of the overall body.

For more flexibility, in addition to the four types listed above, MOMENTS-II possesses the capability of accepting "Known Parts" for which all the required properties are already known (with respect to any coordinate system with origin at the Part's center of gravity). For Known Parts, none of the first type calculations are necessary, but the others are required because their (known) properties must still be translated to the Reference System and combined with those of the other Parts, in order to obtain the desired properties of the overall body.

1.1.3   Rotation and Translation Information

This subsection describes in more detail the Rotation/Translation (Positional) information which is used to specify the location and orientation of each Input System relative to the Reference System. Explanations of the use of Rotation/Translation information in conjunction with both Unknown and Known Parts are presented here.

Rotation information describes the orientation of a Part's Input System relative to the Reference System, and it consists of the nine direction cosines $RM(I,J)$ of the Input Axes (the i, j, and k vectors in the Input System) with respect to the Reference Axes. For each Part, this information is necessary only if its Input Axes are not parallel (respectively) to the Reference Axes; if the two systems are parallel this Rotation information is not necessary. For each Part, the value of an input variable (NR) indicates whether Rotation information is included.

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Translation information describes the location of
the Input System relative to the Reference System, and
it consists of the three Reference System Coordinates
(XP, YP, ZP) of the origin of the Input System. For
Unknown Parts, this information is necessary only if
the origin of the Input and Reference Systems do not
coincide; if they are the same, it is not necessary.
For Known Parts, the user must always specify the
Reference coordinates (CGX, CGY, CGZ) of the center of
gravity of the Part, which must also be the coordinates
of the origin of its Input System. Therefore, this
center of gravity data replaces any extra Translation
information, the latter not permitted for Known Parts.
For each Unknown Part, the value of an input variable
(NTR) indicates whether Translation information is
included; this variable does not apply for Known Parts.

Rotation/Translation information is also useful
in applications requiring more than one computer run,
such as when one would like to use the results of a
MOMENTS-II run to simplify the inputs to a succeeding
run. If an object has been modelled, the input cards
prepared, and a run of the program made, one could easily
re-position any Parts of the object with respect to the
Reference System (for the next run) by merely changing
the appropriate Rotation/Translation information, with-
out having to change any of the Dimensional inputs.

To obtain the properties of an object (which has
already been run through MOMENTS-II) but with respect to
a new Reference System, one may use the initial results
(either with respect to the previous CG or Principal
Axes) as inputs, considering the object in the next
run as a single Known Part, by using the proper Rotation
information. This can be further explained as follows:
among the output from MOMENTS-II are the moments and
products of inertia with respect to the Reference System,
with respect to the CG-System, and with respect to the
Principal Axes. Since the moments and products of inertia
to be supplied for a Known Part must be with respect to
its Input System which must have its origin at the CG
of the Part, these initial results to be used as inputs
for the next run may be either with respect to the previous
"CG-Axes" or with respect to the previous Principal Axes
(both of which have their origins at the CG of the object).
The Rotation information will then be the direction cosines

of either the CG-Axes (these are the same as those of the
old Reference Axes because the two systems are parallel)
or of the Principal Axes with respect to the new Reference
Axes respectively. The former will generally be easier to
find because one may become accustomed to picturing objects
in terms of the original Reference System. These moments
and products, together with the coordinates of the CG
of the Part with respect to the Reference Axes (and
the moments, volume, and mass) locate and describe
the object completely relative to the new Reference Axes
and enable MOMENTS-II to calculate the required properties
relative to these new axes.

1.1.4   Coordinate Systems Used by MOMENTS-II

There are basically four types of coordinate
systems (sets of othogonal axes) which are used repeatedly
by MOMENTS-II. The following outlines the definitions
of each of these:

1.   Reference System - This set of axes pro-
vides a single overall coordinate system with respect
to which all Parts are (indirectly) described. If
one needs to find the properties of a body with respect
to a particular set of axes, these should be considered
as the Reference Axes. Their position is only meaningful
relative to that of the object to be analyzed. The
position of each Part is specified relative to its
Input System by Input System data, and the position of each
set of Input System Axes is specified relative to the fixed
Reference System (i.e., in Reference Coordinates) by Positional
information. Together, these completely describe the
position and orientation of each Part with respect to the
Reference Axes. Aside from this Rotation/Translation
information, only the center of gravity of Known Parts
must be inputted in Reference Coordinates.

The moments and products of the overall body with
respect to the Reference System are printed on the last
page of the program's output. They cannot generally be
used as inputs in treating the object as a Known Part
in later runs because the origin of the Reference System
does not usually coincide with the center of gravity
of the object.

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2. <u>Input Systems</u> - Each Part must be assigned an Input System which is fixed in relation to the Part as restricted in section 1.2 and illustrated in Figures 1 to 5. However the user is somewhat free to position the Input System by specifying the Part's minimum z-coordinate DZ in this system and possibly the lower and upper angles TH(1), TH(2) describing the Part's rotation about this z-axis. The only other restriction is that the Input System of a Known Part must have its origin at the center of gravity of the Part, which is specified with respect to the Reference System.

Rotation/Translation inputs locate the Input Systems relative to the Reference System, but if an Input System coincides with the Reference System, no Rotation or Translation information is necessary. If an Input System is <u>not</u> parallel to the Reference System, Rotation information <u>is</u> necessary for that Part and if the origin of an Input System does <u>not</u> coincide with the origin of the Reference System, Translation information is necessary. If both of these conditions are present, then both Rotation and Translation information (in that order) are required for the Part.

3. <u>CG-System</u> - This system is simply a set of axes which are parallel (respectively) to the Reference Axes but have their origin at the center of gravity (CG) of the entire object being analyzed. The moments and products of inertia of the object computed in this system are included in the program's output and can be used (they are probably the easiest to use for this purpose) as inputs in treating the same object as a Known Part in succeeding runs of MOMENTS-II.

4. <u>Principal Axis System</u> - This system has its origin at the center of gravity of the overall object; two of the axes in this system are defined in the directions of maximum moment and minimum moment and the third direction is chosen so as to form a (right handed) orthogonal coordinate system.

The principal moments are also included in the output from the program; they are actually the moments of inertia with respect to the Principal Axis System. The products of inertia with respect to this system are by definition zero. These moments can also be used as inputs in treating the object as a Known Part in succeeding computer runs.

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1.2 TYPES OF PARTS

This section describes the four types of Unknown Parts which are shown with their Input Systems in Figures 2-5, as well as a fifth type, the Known Part. This includes further explanations of the Dimensional and Input System information necessary to describe each type of Part. The actual FORTRAN names of the inputs are shown on the figures and the format and order of the input cards used to feed this information to the program are described in the Input Guide, section 1.3.

1.2.1 Basic Parts

The first type of solid acceptable to MOMENTS-II is called a Basic Part; it is defined as a collection of one or more adjacent sectors (conical wedges), each taken from a (possibly different) right circular frustum of a cone whose axis defines the Input System z-axis of the Basic Part. Even though a single right circular frustum of a cone is symmetric about its axis, the Basic Part, being a combination of different sized sectors, will not usually be symmetric about its Input System z-axis (the common axis of all its sectors). These individual sectors of frusta of cones are the building blocks of all Basic Parts; for this reason they will be referred to as "Prototype Sectors".

There are five allowable kinds of Basic Parts (indicated to the program by the value of NTYPE), the simplest of these consisting solely of a single "Prototype Sector". This kind is sometimes referred to as a "Floating Sector" because it requires the specification of both its lower and its upper angles (its lower angle is "free-floating") rather than requiring the lower angle to be 0° as is the case for the other four kinds of Basic Parts. A Prototype Sector is pictured in Figure 1; its size can be described by its length H, and its radii R1(1) and R2(1) at the minimum Input System z-coordinate and maximum Input System z-coordinate ends respectively. The Input System z-axis of a Prototype Sector must be chosen to coincide with the axis of the sector; the Part's position along this axis is specified by DZ (the minimum Input System z-coordinate of any point on the Part). The relative position of the Part with respect to the x and y Input

Axes is specified by the lower and upper angles, TH(1) and TH(2), which are measured from the positive xz-plane (at 0°) in a positive direction about the z-axis towards the positive yz-plane (at 90°). These, together with the density (DEN) and the fact that it is a Floating Sector (NTYPE=1), completely describe the Dimensional and the Input System data for this type of Basic Part.

In the special case R1(1) = R2(1) = R, the Prototype Sector is a sector of a (right circular) <u>cylinder</u> rather than of a cone and its (constant) xy-plane cross section is a sector of a circle of radius R. If R1(1) $\neq$ R2(1), this cross section varies with z, but it is always a sector (of a circle) whose radius R(z) depends linearly on z (and is given by Eqn. 2c). It should be remembered that although a Prototype Sector is in general a piece of a (right circular) cone which is symmetric about the z-axis, the sector itself (unless TH(2) - TH(1) = 360°) is <u>not</u> symmetric about this axis.

The other four kinds of Basic Parts (indicated by NTYPE = 0, 2, 3 or 4) are simply combinations of more than one Prototype Sector. Each of the sectors of a Basic Part may be assigned a different density, different angular measure and different radii, but all of the sectors of a single Basic Part are assumed to have the same minimum z-coordinate (DZ) and length (H), and a common axis which defines the Input System z-axis. One must always supply as input the radii of each sector, and there are four other kinds of Basic Parts to account for the variation or non-variation of the density and/or angular measure from sector to sector (which are inputted only when necessary). In contrast to the Floating Sector, here the lower angle of the first sector is assumed to be 0°, so that because the sectors are assumed to be contiguous, the necessary inputs (if the sector angular size is to vary) are simply the <u>upper</u> angles of the sectors, in order. Further descriptions of these four kinds of Basic Parts can be found in the Input Guide, Section 1.3.

1.2.2 Standard Wedges

The term Standard Wedge will refer to a (possibly truncated) right angle wedge (or trapezoidal prism) as illustrated in Figures 3a and 3b. Its dimensions are its length H, its width W and its heights RR1 and RR2 at the

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lower-z and upper-z ends respectively. The relation of a Standard Wedge to its Input System is defined by requiring that its base be in the yz-plane and that the xz-plane divide the wedge in half. The only freedom the user has in choosing the Input System of a Standard Wedge is the choice of DZ. This together with the density (DEN) and the fact that it is a Standard Wedge completely describe the Dimensional and Input System data for this Part.

The actual appearance of the Standard Wedge is of two forms according as either RR1 = 0 or RR2 = 0, or as both are non-zero. In the former case the Part should correctly be called a "right angle wedge" or right triangular prism (Figure 3b) whereas in the latter case it should more properly be called a "truncated wedge" (or trapezoidal prism) since in that case one vertex is sliced off by a plane parallel to the xy-plane (and as a result the Part has six faces instead of the five for a "right angle wedge").

1.2.3 Angular Wedges

The term Angular Wedge will apply to a wedge (or triangular prism) positioned as illustrated in Figure 4. Its dimensions are its length H and its lower "radius" RR1 and upper "radius" RR2. The relation of an Angular Wedge to its Input System is defined by its lower and upper angles TH(1) and TH(2) and by DZ. These together with the density (DEN) and fact that it is an Angular Wedge completely describe the Dimensional and Input System data for this type of Part.

Angular Wedges have a constant xy-plane cross section (it does not vary with z) which is a triangle with sides of length RR1 and RR2 and an included angle measuring TH(2)-TH(1) (which of course must be less than 180°). This is in contrast to a Prototype Sector which has only one radius at each end but may vary in size from end to end.

It should be noted that Angular Wedges are actually a combination of one or two Standard Wedges which have been rotated and translated. They are included as a separate type of Part to relieve the user of the task of constructing

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them himself. The program actually performs these rotations and translations and computes the properties of the Angular Wedge by properly combining the properties (which it computes) of the Standard Wedges of which the Angular Wedge is composed. A more detailed description of this construction of Angular Wedges can be found in sections 2.4 and 3.5.

1.2.4 Concave Parts

The term "Concave Part" will be applied to a body shaped like the one illustrated in Figure 5a. It is similar to the Angular Wedge described in 1.2.3, differing only in that the surface opposite its interior angle, of magnitude $(TH(2) - TH(1))$, is not a plane but rather is a part of the surface of a right circular cylinder whose axis is a line parallel to the z-axis (but is outside the Part itself), and is located so that the resulting Part is <u>not</u> convex. That is, as in Figure 5b, O´and O do not lie <u>on</u> the same side of line P_1P_2. Its dimensions are its length H and its lower "radius" $R1(1)$ and upper "radius" $R1(2)$.

Its relation to is Input System is defined by its lower and upper angles $TH(1)$ and $TH(2)$ and by its minimum z-coordinate DZ, just as in the case of the Angular Wedge. The Concave Part has a constant xy-plane cross section (Figure 5b) which is a closed figure consisting of two lines of lengths $R1(1)$ and $R1(2)$ making angles of $TH(1)$ and $TH(2)$ respectively with the xz-plane and an arc which is part of a circle with center at the point (XC, YC).

It should be noted that a Concave Part is a combination of two Angular Wedges minus a Prototype Sector, rotated and translated together. The program constructs and computes properties of Concave Parts by properly combining results of calculations on Angular Wedges and Prototype Sectors. This is illustrated in Figure 5b and is further explained in 2.5 and 3.6. As in the case of Angular Wedges, this type Part is included, even though it is not independent of the previous types of Parts, as a convenience to the user.

1.2.5 Known Parts

Whereas MOMENTS-II is capable of calculating properties of the type Parts described in 1.2.1 - 1.2.4, this ability is needed specifically to enable the program to achieve its main purpose which is to combine them

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to obtain the properties of the entire composite object. If a body is composed of (possibly irregularly shaped) pieces whose properties are already known, in addition to the pieces which can readily be approximated by Parts, the program must also be able to fulfill its main purpose of finding the properties of the entire body. For this reason, in addition to the four "Unknown Parts" previously discussed, MOMENTS-II will accept "Known Parts". A Known Part is simply an object whose shape need not be considered but whose properties and location must be described to the program. As for the other Parts, each Known Part must be assigned an Input System; in this case. however, the origin of the Input System must be the center of gravity of the Known Part, although its orientation (of its Input System) may be arbitrarily chosen by the user, as before.

In addition to the "invariant properties", namely the volume (V) and mass (AMASS), the user must supply the "relative properties" I_{xx}, I_{yy}, I_{zz}, I_{xz}, I_{yz}, I_{xy} (see 1.1.1 - their FORTRAN names are XX, YY, ZZ, XZ, YZ, XY) relative to the Input System.

The location of the Input System is specified by giving the Reference System coordinates of the center of gravity of the Part (CGX, CGY, CGZ - which are also the coordinates of the origin of the Input System) so that none of the usual Translation information is needed or even permitted.

As before, Rotation information may be used to describe the orientation of the Input System relative to the Reference System for Known Parts also.

These restrictions, which in essence require that the moments and products of inertia for a Known Part be known relative to any right-handed orthogonal coordinate system with origin at the center of gravity of the Part, should not increase the difficulty of using the program, since it is natural that these properties, if known, will be relative to such a coordinate system.

A second function of this type of Part stems from the following consideration: although once an object has been "modelled" into Parts and punched onto cards it can be inputted in that form in the future, it would

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be simpler if one could easily utilize the results of
the first calculation, rather than re-using all the
input cards and requiring the program to re-calculate
and re-combine the properties of these Parts each time
the identical object is to be used again. This can be
accomplished by using the Known Part feature, as follows:
if an object (represented by a group of Parts) is to be
combined with other objects in the future, the output
of the initial MOMENTS-II run can be punched onto (two
or three) cards as a single Known Part to be used as
inputs for the future runs (thus avoiding duplicate
calculations and saving time).

A third but related use of the Known Part feature
is that it allows one to find the properties with respect
to a new Reference System of an object which has been
modelled and inputted to MOMENTS-II previously with respect
to an original Reference System, by merely adding the
proper Rotation information and the proper center of
gravity coordinates to the results already obtained, and
by treating the object as a Known Part. This is in
contrast to keeping the complete original set of inputs
for the Parts making up the object and adding the proper
Rotation/Translation information to each Part's input.
This point was mentioned in the discussion of Rotation/
Translation information, sub-section 1.1.3.

1.3 INPUT GUIDE

This section 's an Input Guide whose purpose is to exhibit the method of preparing the input cards required by MOMENTS-II. Subsection 1.3.1 outlines the procedure to follow in using the program to analyze (find the properties of) an object, 1.3.2 lists the actual input cards and input formats and 1.3.3 explains these input cards in more detail. The sample cases in 1.5 complement this discussion and should be examined before attempting to use MOMENTS-II.

1.3.1 Procedure (Input Preparation)

This is an outline of the steps to follow in using MOMENTS-II to analyze a particular object.

1. While viewing an illustration or blueprint of the entire object, choose a set of Reference Axes, relative to the object, on the basis of convenience or (more often) because it is desired to find the moments and products of the body with respect to these axes.

2. Divide the body into sections, which are merely portions of the object which can be conveniently modelled individually into Parts. A section might be the region of space between two parallel planes containing a portion of the object, such that the cross section (in a plane parallel to these) is constant in this region. This is merely a convenience and its use will be evident in Sample Case 3.

3. Model each section by deciding how many Parts will be used to approximate the section and exactly what each of these Parts will consist of. (Note that every point in the body must be in at least one Part but may appear in more than one because of the use of Parts with negative densities for deleting material.)

4. a) For each Part, decide whether to classify it as a Basic Part, Standard Wedge, Angular Wedge, Concave Part or Known Part, and choose an (optional) identification or name for each Part.

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b)   Choose a convenient set of Input Axes for each Part according to the restrictions in 1.2 by specifying the Input System information.

c)   Find the Dimensional information needed for each Part.

d)   Prepare the Rotation/Translation information for each Part (this depends on the choice of Input Axes relative to the Reference Axes) as explained in 1.1.3.

5.   Punch all the input cards in accordance with the tabulation of input cards in 1.3.2 using the data prepared as explained above.

1.3.2 Input Cards for MOMENTS-II

*1. Title Card

(TITLE(K), K=1,8)   (8A10)

Any alphanumeric title may be placed in columns
1-80 and it will be printed at the top of every page of
output.

*2. Option Card

DEN1, NJ1, NPRNT, N1, N2, N3, N4   (F10.5,7I5)

DEN1   Default density
NJ1   Default number of sectors/Basic Part
            ⎧0 Output type = Summary (first and
            ⎪   last pages only)
NPRNT =   ⎨1 Output type = Partial (omit Basic
            ⎪   Part radii)
            ⎩2 Output type = Complete
N1   Number of Basic Parts
N2   Number of (Standard and Angular) Wedges
N3   Number of Concave Parts
N4   Number of Known Parts

3. Basic Part Cards

** a)   IDENT, H, DEN, DZ, NTYPE, NJ, TH(1), TH(2),
         NR, NTR
                              (A10,3F10.4,2I5,2F10.3,2I5)

** b)   (R1(J),J=1,NJ)   (8F10.4)
** c)   (R2(J),J=1,NJ)   (8F10.4)
    d)   (TH(J),J=1,NJ)   (8F10.4)
    e)   (RH),J=1,NJ)   (8F10.4)
    f)   ((RM(I,J),I=1,3),J=1,3)   (9F8.5)
    g)   XP,YP,ZP   (3F10.4)

One set of a-c is required for each Basic Part. The
other inputs d,e,f and g are optional and are included
or not depending on NTYPE, NR, and NTR.

*   These cards are required once for each case.
** These cards are required once for each Part.

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IDENT	Optional (10 character maximum) identifying name for Part
H	Length of Basic Part (measured parallel to the Input System z-axis)
DEN	Density of material in Basic Part
DZ	Minimum z-coordinate of Basic Part (with respect to Input System)

$$NTYPE = \begin{cases} 0 & \text{Normal Basic Part with NJ equiangular sectors} \\ 1 & \text{Floating Sector from lower angle TH(1) to upper angle TH(2)} \\ 2 & \text{Variable angles for sectors, density constant. TH(·) inputs to follow.} \\ 3 & \text{Variable density for sectors, angle size constant. RH(·) inputs to follow.} \\ 4 & \text{Variable angles and densities. TH(·) and RH(·) inputs to follow.} \end{cases}$$

NJ	Number of sectors in the Basic Part
TH(1), TH(2)	Lower and upper angles of Floating Sector (degrees). Relevant for NTYPE=1 only.

$$NR = \begin{cases} 0 & \text{Input Axes are parallel to Reference axes.} \\ 1 & \text{Input Axes are \underline{not} parallel to Reference Axes. Rotation information to follow.} \end{cases}$$

$$NTR = \begin{cases} 0 & \text{Origin of Input System coincides with origin of Reference System.} \\ 1 & \text{Origin of Input System does \underline{not} coincide. Translation information to follow.} \end{cases}$$

R1(J)	Radii of sectors at lower-z end of Basic Part.
R2(J)	Radii of sectors at upper-z end of Basic Part.
TH(J)	Upper angles of sectors. Included if NTYPE=2 or 4 only. (Sectors are assumed contiguous with lowest angle equal to zero.)
RH(J)	Densities of sectors. Included if NTYPE=3 or 4 only.
RM(I,J)	Rotation information. Three unit vectors in directions of the Input System x, y, and z axes respectively, expressed in Reference coordinates. Included if NR=1 only.
XP, YP, ZP	Coordinates of the origin of the Input System, expressed in Reference Coordinates. Included if NTR=1 only.

4. Standard Wedge Card(s)

**a) IDENT, H, DEN, DZ, RR1, RR2, W, blank, NR, NTR

                                               (A10,5F10.4,2F8.3,2I2)

b) ((RM(I,J),I=1,3),J=1,3)   (9F8.5)

c) XP,YP,ZP   (3F10.4)

IDENT,H,DEN,DZ,NR,NTR,RM,XP,YP,ZP are as in 3.

RR1,RR2 Heights at lower and upper-z coordinate
            (ends) of Wedge

W   Width of Standard Wedge (measured parallel
            to Input System y-axis)

Field of width 8 after W must be blank to indicate
Standard Wedge rather than Angular Wedge.

5. Angular Wedge Card(s)

**a) IDENT, H, DEN,DZ,RR1,RR2,TH(1),TH(2),NR,NTR

                                               (A10,5F10.4,2F8.3,2I2)

b) ((RM(I,J),I=1,3),J=1,3)   (9F8.5)

c) XP,YP,ZP   (3F10.4)

IDENT,H,DEN,DZ,NR,NTR,RM,XP,YP,ZP are as in 3.

RR1,RR2   Lower and upper radii of Angular Wedge

TH(1),TH(2) Lower and upper angles of Angular Wedge
            TH(2) > 0.0 is required.

6. Concave Part Cards

**a) IDENT,H,DEN,DZ,R1(1),R1(2),TH(1),TH(2),NR,NTR

                                             (A10,5F10.4,2F8.3,2I2)

**b) XC,YC   (2F10.4)

c) ((RM(I,J,I=1,3),J=1,3)   (9F8.5)

d) XP,YP,ZP   (3F10.4)

IDENT,H,DEN,DZ,TH(1),TH(2),NR,NTR,RM,XP,YP,ZP are
as in 5.

R1(1),R1(2)   Lower and upper radii of Concave Part.

XC,YC   x and y Input System coordinates
            of center of exterior cylinder
            used to define Concave Part.

7. __Known Part Cards__

   **a)   VOL,AMASS,CGX,CGY,CGZ,IDENT,NR   (5F10.4,A10,I5)
   **b)   XX,XY,YY,XZ,YZ,ZZ   (6F10.4)
     c)   ((RM(I,J),I=1,3),J=1,3)   (9F8.5)

IDENT and NR are as previously defined.

VOL	Volume of Known Part
AMASS	Mass of Known Part
CGX,CGY,CGZ	Coordinates of CG of Known Part, in Reference Coordinates
XX,YY,ZZ	Moments of Inertia of Known Part with respect to its Input System
XY,XZ,YZ	Products of Inertia of Known Part with respect to its Input System

(The Input System of a Known Part must have its origin at the CG of the Part.)

To _delete_ a Known Part, use negative AMASS only; to delete any other (Unknown) Part, use negative DEN instead. The products, moments and volume for a Known Part should _not_ be reversed in sign if the Part is to be deleted; MOMENTS-II will do this automatically.

The TITLE and OPTION CARDS are followed by _all_ the BASIC PART CARDS, all the WEDGE CARDS, _all_ The CONCAVE PART CARDS, and all the KNOWN PART CARDS, in that order. In all cases, The Rotation/Translation cards are placed directly after The Input Cards for The Part they refer to.

1.3.3 Discussion of Input Cards

1. TITLE CARD

This card must be included as the first card for each set of data relating to a single object or body (i.e., for each case). The alphanumeric data appearing on this card will be read and printed (exactly as punched) on the top line of each page of output. If no heading is desired, a blank card should be used.

2. OPTION CARD

This card must be included as the second card for each case. The optional variables DEN1 and NJ1 are default values for the density (DEN) for all Unknown

Parts and for the number of sectors (NJ) in a Basic
Part respectively. If for any Part the value of either
DEN or NJ is read in as zero (or blank), the corresponding
default value read from the OPTION CARD will be used
in place of the zero. If DEN1 is also zero in such a
case, the value used for the density will be zero, in
which case an incorrect volume (non-zero) will probably
be calculated.

The variable NPRNT is used to specify the amount
of information to be printed. Setting NPRNT=0 will
suppress all output except for the first page (summary
of the options) and the last page (properties of the
entire body). Setting NPRNT=2 will allow the complete
output to be printed. NPRNT=1 will have the same effect
as the latter case except that a slightly shortened
form of output will be used for Basic Parts, which does
not include a listing of any input arrays of variable
radii, angles and densities for the sectors. This
option will be explained further in 1.4.

The input variables N1, N2, N3 and N4 tell the
program how many of each type Part will be included in
the current case.

3.   BASIC PART CARDS

One set of cards in set 3 is required for
each Basic Part to be used. Each Part may be assigned
an (optional) identifying name consisting of any 10 (or
less) characters. This IDENT is simply for the convenience
of the user and since it is included in the output it
allows one to find a particular Part by name. The vari-
ables DZ and H are standard inputs which correspond
to quantities in Figure 1. In all cases the sectors in
a single Basic Part are assumed to have a common length
H, and NJ specifies the number of sectors in the Basic
Part. (The current limit of 72 can easily be changed.)

The input variable NTYPE informs the program
which of the five kinds of Basic Parts the current one
is, i.e., whether or not the angles and/or densities
will vary from sector to sector in the Part. The five
cases are considered individually as follows:

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If NTYPE=0, the program assumes that the Basic Part consists of NJ sectors each with an angular measure of 360/NJ degrees and density DEN, and that they are situated such that the first section has lower and upper angles of 0° and 360°/NJ respectively, and continuing in a contiguous fashion, the last sector having lower and upper angles of (360° - 360°/NJ) and 360° respectively. Except for their varying angular positions, these sectors may differ in size, only because of varying "left" and "right" end radii variation. These radii are read in as R1(J) and R2(J), for J=1,...,NJ, on cards 3b and 3c as is the case for all Basic Parts. For this kind of Basic Part, cards 3d and 3e should be omitted.

If NTYPE=1, the Basic Part is assumed to be a Floating Sector for which the inputs TH(1) and TH(2) specify the angular position (and angular measure) as in Figure 1. (These variables are ignored for other values of NTYPE.) For this kind of Basic Part NJ must be 1 and Cards 3d and 3e must be omitted.

If NTYPE=2, the program assumes that the angular measure of the NJ sectors is allowed to vary, so that the (upper) angles must be inputted using card type 3d. Here, the lowest angle of the first sector is assumed to be zero and the NJ angles which are inputted are considered to be the upper angles of each sector. Since the sectors are assumed to be contiguous the lower angle of the J+1st sector is defined to be the upper angle of the Jth sector, so that these upper angles define the angular boundaries completely and uniquely. For example, NJ=4 and TH(1),...,TH(4) equal to 24°, 72°, 168°, 360° would be interpreted as describing four sectors whose angular ranges are 0° to 24°, 24° to 72°, 72° to 168° and 168° to 360°; they are sectors of angular measure 24°, 48°, 96°, and 192° respectively. To input the last three of these sectors only, that is, to input three sectors of varying angular measure with first sector not having lower angle zero, one could follow the above procedure, but also input R1(1)=0 and R2(1)=0 by punching 0.0 in the first fields of cards 3c and 3d respectively. (For any value of NTYPE, this method of setting radii equal to zero rather than setting densities equal to zero should be used if one needs to skip certain sectors.)

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If NTYPE=3, the program assumes that the density
will be allowed to vary from sector to sector and that
these densities will be inputted on card set 3e, one
density for each sector. Card set 3d should be omitted,
because the sectors will be assumed equiangular as if
NTYPE=0.

If NTYPE=4, it will be assumed that both the
angles and the densities will vary by sector, and there-
fore, card sets 3d and 3e must be included to supply
this information. The angles are then handled exactly
as if NTYPE=2 and the density will be handled as if
NTYPE=3.

In general, the density is handled as follows:
For NTYPE=0,1 or 2 the constant density DEN from the
preceding card 3a is used for all sectors. For NTYPE=3
or 4 the density of the individual sectors are specified
in the input array RH(J) on the card set 3e. In these
cases the value of DEN (card 3a) will be used as a
default value for the sectors in the same Basic Part,
and if DEN is zero the value of DEN1 will be used in
its place even if that also is zero.

As for the other three (Unknown) Parts, the indi-
cators NR and NTR inform the program if any Rotation or
Translation data (respectively) is included. This in-
formation always appears as the last card(s) for the Part
to which it applies; for Basic Parts they are card 3f
and 3g. Card 3f, as for all other Rotation cards, con-
tains nine entries. The first three are the Reference
System Components of the Input System x-axis direction,
the next three are the Reference System Components of
the Input System y-axis direction and the last three
are the Reference System Components of the Input System
z-axis direction. (The matrix of these nine elements
must be orthonormal.) Card 3g, as for all other Trans-
lation cards, contains simply the Reference System
Coordinates of the origin of the Input System.

To summarize the Basic Part cards, card sets 3a,
3b and 3c are required for each Basic Part, card sets
3d and 3e depend on NTYPE for inclusion or exclusion,
and cards 3f and 3g depend on NR and NTR. The formats
of the data are indicated in parentheses next to each

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card set. Card sets a, f and g consist of one card each and card sets b, c, d and e contain as many cards as are necessary to specify one quantity for each of the NJ sectors; since these quantities are punched 8 per card, the number of cards in any of these sets for a given Basic Part is the greatest integer in (NJ+7)/8, where NJ represents the number of sectors in that Part.

4. STANDARD WEDGE CARDS

The entries on card type 4a are analogous to the previously explained variables except that RR1 and RR2 are the two heights of the Wedge at the lower-z and upper-z ends respectively and W is the width of the Wedge. The eighth field on data card 4a should be blank in order to inform the program that the wedge is a Standard Wedge rather than an Angular Wedge. Cards 4b and 4c contain the Rotation and Translation information, their inclusion depends on NR and NTR as before.

5. ANGULAR WEDGE CARDS

These cards are identical to those in 4., except that W does not apply but instead the angles TH(1) and TH(2) (see Figure 4) are required. Here, TH(2) must be greater than zero in order to differentiate Angular Wedge cards from Standard Wedge cards. (Thus, to input an Angular Wedge with upper line in the xz-plane, one would set TH(2) equal to 360° rather than equal to zero.) As should be obvious, the angular measure of a sector, given by TH(2)-TH(1) or by TH(2)-TH(1)+360° if the first result is negative (for example if TH(1)=300°, TH(2)=30°), must be less than 180°, since it is actually an angle of a triangle. It is not required that Standard Wedge cards precede Angular Wedge cards, as these are treated similarly (for input purposes) by MOMENTS-II.

6. CONCAVE PART CARDS

The first card (6a) contains the usual Dimensional and Input System data as well as the IDENT and Rotation/Translation indicators NR and NTR. The inputs R1(1) and R1(2) are the lower and upper radii making angles

of TH(1) and TH(2) degrees respectively with the xz-plane. Card 6b contains the Input System Coordinates XC, YC of the center of the (exterior) cylinder necessary to define the Concave Part. It should be remembered that the axis of this cylinder must be parallel to the Input System z-axis of the Part. The radius of the cylinder is not needed as input, since MOMENTS-II computes this quantity.

7. KNOWN PART CARDS

These cards must supply the properties of the Known Part relative to its Input System, whose position relative to the Reference System is specified by the inputs CGX, CGY, CGZ and by the Rotation information as explained in 1.1.3. As mentioned previously, the variables CGX, CGY and CGZ are the Reference Coordinates of the center of gravity of the Part which is also the origin of its Input System.

The order of the input cards follows the order they are listed in 1.3.2. For each set (3, 4, 5...) of cards describing a single Part, cards (a, b, c...) in the set appear in the order listed above, with the Rotation/Translation cards (if necessary) always appearing at the end of the data for a particular Part. The order of card sets describing like types of Parts is arbitrary, e.g., if one has two card sets describing Basic Parts, it makes no difference which of these two sets appears first. The order of card sets describing different types of Parts must, however, be as in the list (1.3.2), with the only exception being that all Wedge cards, both Standard and Angular, may be treated as a single type of Part, the Wedge, for which the preceding statement applies. The structure of an input deck (for a single object) is as follows: One Title Card and one Option Card must appear as the first and second cards respectively. Following these, one inserts all Basic Part Cards followed by all Wedge Cards followed by all Concave Part Cards, and finally followed by all Known Part Cards. If more

than one object is to be analyzed in a single run of MOMENTS-II, each input deck is constructed as above and the decks are placed sequentially by case (no separators of any kind are necessary), each one beginning with the required Title and Option Cards. Cards labelled (*) in 1.3.2 are required once for each case, and cards labelled (**) are required once for each Part of the type to which they apply.

In all cases angles are measured in a (positive) rotation about the z-axis, with a line in the positive xz-plane measured at 0° and a line in the positive yz-plane at 90° etc. One should always use positive angles between 0° and 360° to express angles. If a floating Sector, Angular Wedge or Concave Part cuts across its Input System xz-plane, it is permissible to define the angles TH(1) and TH(2) such that TH(1) > TH(2), e.g., TH(1) = 330° and TH(2) = 20°. (Although other possibilities such as TH(1) = -30° and TH(2) = 20° may also execute successfully for such a case, it is not advisable to input angles in this fashion.)

Finally, as is generally the case, if any optional variable is omitted, its field as specified by the appropriate format appearing in 1.3.2 must be left blank; no compaction of data on a card should be attempted. For optional cards, however, the whole card should be omitted (and not replaced by a blank card) if the data on the card is not required.

1.4 OUTPUT GUIDE

The purpose of this section is to explain all the
information printed as output from MOMENTS-II. The
four types of data provided as outputs are the options,
the inputs for all types of Parts, the computed properties
of all types of Parts, and the computed properties of
the entire body.

1.4.1 MOMENTS-II Option Page

The first page of output from MOMENTS-II will be
referred to as the Option Page. The title, exactly as
appearing on the Title Card, is printed at the top of
this page as on all other pages. The Option Page is al-
ways included in the output and it lists precisely the
information contained on the Option Card. The values
of the input quantity NPRNT are indicated on this page
by specifying the output type as SUMMARY, PARTIAL, or
COMPLETE. COMPLETE indicates that all outputs are printed,
and SUMMARY indicates that only the Option Page and the
last page (PROPERTIES OF THE ENTIRE BODY) are printed.
It will be pointed out in 1.4.2 which print-outs are
omitted or modified if the PARTIAL type is chosen;
this option only affects output for Basic Parts.

1.4.2 Listing of Inputs

For input/output purposes, both Standard Wedges
and Angular Wedges are considered simply as Wedges. Their
input/output formats are almost identical and are combined.
Therefore, the output for Unknown Parts is divided into
three sections, for Basic Parts, Wedges and Concave
Parts. Furthermore, each of these sections consists of
a listing of input data (not card images) and a listing
of output data. The following summarizes these as has
been done in 1.3.2 for Input cards.

1. Basic Parts - On the section entitled "INPUT
DATA - BASIC PARTS", there is at least one main line of
output for each Basic Part, which gives the Part a number
and lists all data which was inputted on Card 3a. For
Floating Sectors (NTYPE=1), the lower and upper angles
(TH(1) and TH(2)) are listed; if they do not apply
(NTYPE=1) their fields each contain a single asterisk (*)

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only. For NPRNT=2 (COMPLETE output), the next group
of lines will contain the lower-z and upper-z end
radii $R1(J), J=1...,NJ$ and $R2(J), J=1,...,NJ$. The
remaining lines will be those which apply for the parti-
cular value of NTYPE being considered. In all cases,
the output for each Basic Part will follow (in order) the
input cards 3a-3g which apply. Thus, if Rotation and
Translation data were inputted, these will be outputted
on the last two lines for the Basic Part. For the
PARTIAL output, which is useful for Basic Parts which
consist of complete frusta of cones rather than single
sectors (i.e., they are symmetric about some axis)
so that the first lower-z and upper-z radii (at either
end of the first sector in the Basic Part) are the only
radii, these radii are listed on the main line in addition
to the standard quantities inputted on card 3a. The
lines corresponding to data inputted on cards 3b-3e are
not printed out, but those corresponding to Rotation/
Translation information are printed whenever applicable
unless NPRNT=0.

2. Wedges - The section entitled "INPUT DATA -
WEDGES", contains the inputs for both Standard Wedges
and Angular Wedges. The data printed out is merely
the information from input cards 4a, b, c and 5a, b, c
in the order the data was inputted to MOMENTS-II. As
usual, one main line of data is printed for each Part,
containing the information on cards 4a or 5a, and the
Rotation/Translation information is printed directly
following the line to which it applies. The program
will also print "STD" or "ANG" in the column labelled
"WTYPE" to distinguish between Standard and Angular
Wedges respectively. This will also be indicated by
the presence of the asterisk (*) in the WIDTH column
(for Angular Wedges) or in the TH(1) and TH(2) columns
(for Standard Wedges).

3. Concave Parts - The next section lists the
inputs for Concave Parts which correspond to the
data inputted on card sets 5a, 5b and also on 5c and/or
5d when applicable. The coordinates XC, YC on card
type 5b are printed last on this main line and are
called "X-CENTER" and "Y-CENTER".

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4. Known Parts - Each Known Part has a main
line printout containing all the information from card
set 6a, b. As before, data from card set 6c is
printed on the next line if it applies.

In all cases, the presence or absence of the
Rotation/Translation lines at the end reveals the values
of NR and NTR, and no other listing of these indicators
is printed.

1.4.3 Listing of (Computed) Properties of Parts

This group of outputs contains sections for
each of the four types of Parts (Basic, Wedges, Concave,
Known) considered as outputs. The outputs for Basic
Parts, Wedges, and Concave Parts all consist of single
lines containing the number of the Part (corresponding
to that used for each Part in 1.4.2, numbers beginning
at 1 for each of the four types of Parts), the IDENT,
and the computed volume, mass and center of gravity in
Reference Coordinates. The "PROPERTIES OF KNOWN PARTS"
section lists the number and IDENT of each Part as well
as the moments and products of inertia with respect to the
Reference Axes. This is in accordance with the practice
of listing quantities which were computed by MOMENTS-II
in this section. (It is this feature that enables one
to input either a number of Known Parts with respect
to a different Reference System or a single Part with
respect to different Reference Systems, to find the
moments and products of either all the objects with
respect to a new Reference System or of a single Known
Part with respect to an arbitrary set of Reference
Systems as described by Rotation cards and the variations
of CGX, CGY, and CGZ.)

1.4.4 Properties of the Entire Body

The last page of output contains information
on computed properties of the entire object. The "MASS"
is merely the algebraic sum of the masses of the Parts
making up the object. The "POSITIVE VOLUME" is the sum
of the volumes of all Parts with positive masses or
densities and the "NEGATIVE VOLUME" is just the sum of
the volumes of the Parts which have been deleted (by the
user). The algebraic sum of these two quantities is
referred to as "NET VOLUME", and it will correspond to
the true volume only if the inputting and modelling have
been done according to certain rules (see note in Sample
Case 1).

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The "COMPONENTS OF THE INTERIA TENSOR WITH RESPECT TO REFERENCE AXES" are just the algebraic sum of the moments and products of all Parts with respect to the CG-Axes (parallel to the Reference Axes but with origin at CG of the object). These are the most useful values for inputting to future MOMENTS-II runs. Finally, the "PRINCIPAL MOMENTS" and "PRINCIPAL AXES" are the eigenvalues and eigenvectors of the inertia tensor matrix with respect to the CG-Axes, and they are the final output on the "PROPERTIES OF THE ENTIRE BODY" page.

1.5   SAMPLE CASES

This section contairs four sample cases to il-
lustrate the use of MOMENTS-II in finding properties of
certain solids.   Each sample case serves a specific
purpose which is mentioned below.   The cases treated
here by no means exhaust the possible applications of
the program.

1.5.1   Sample Case 1

The following sample case consists of a hypo-
thetical and greatly oversimplified shell.   It is the
same as that treated in Ref 3 (describing the original
MOMENTS program) and it is included here to illustrate
the simplicity of using the TRANSL information (rather
than the approximations used previously) to input a Part
whose Input System is parallel to the Reference System
but has its origin at a different point.

The sample body is illustrated in Figure 6 and it
has already been divided into sections as described in
1.3.1.   Much of the following description of the object
has been taken from Reference 3.   Such a case might
be considered to investigate the effect on its properties
of shifting a piece of a previously symmetric object, so
as to make it asymmetric.

Section 1 and 2 are already special cases of frusta
of symmetric cones, so that each is treated as a Basic
Part.   The third section labelled "Known" is identical
to the second, but will be inputted as a Known Part to
illustrate the use of Known Part cards.   For this Part
the moments and products will be inputted rather than
requiring the program to calculate them.   The axes of
these three Parts all coincide with the Reference z-axis,
so their Input Systems are all chosen parallel to the
Reference System and since DZ will be used to account
for any z-displacement the two systems are identical and
no Rotation or Translation is necessary for them.

The fourth section contains a cylindrical core of
material which is denser (density =0.5) than the rest
of the section (density = 0.3).   To input this section
three Parts are used: first the basic section, a cylinder
with density .3 as a Basic Part is inserted, next the

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material occupying the space where the core is to be placed is considered as a Basic Part to be deleted (using density = -0.3, to indicate a deletion of material of density 0.3), and finally the core (density = 0.5) is inserted as another Basic Part. These last two Parts are identical in shape but differ only in sign and magnitude of density.

If one represents this section as two Parts instead of three by simply adding the difference (0.2=0.5-0.3) between the core density and the basic density as Part 4 (not requiring Part 5), correct results would be obtained for all quantities except for volumes. The "POSITIVE VOLUME" would be the same, but the "NEGATIVE VOLUME" would be less negative, thus resulting in a "NET VOLUME" which is greater than the true volume. To insure that the "NET VOLUME" actually represents the true volume, one always must model an object as if one were constructing it Part by Part. That is, one must not insert a Part into a region of space already occupied by another Part without deleting the Part already occupying that space (by assigning a negative density of equal magnitude to another Part otherwise identical with the first).

The fifth section or tail section is inputted as two basic Parts. The first is a frustum of a cone (density =0.2) which actually includes more than the tail section. To delete the cone-shaped extra piece on the end, another Part is created and assigned a negative density of -0.2.

Thus a total of seven Basic Parts (all NTYPE=0) and one Known Part are used to describe this simplified shell. It should be noted that in contrast to the method used in Reference 3, the core, which is symmetric about an axis parallel to and 0.5 units above the z-Reference Axis, is represented exactly here and is not approximated. This is made possible by choosing an Input System for the two Parts (deletion and insertion) describing the core, with origin at the point (0.5, 0.0, 0.0), indicated by including for each a TRANSL card with XP=0.5, YP=0.0, and ZP=0.0 and a value of 1 for NTR. A comparison to the method of Reference 3 will show the amount of work saved by this procedure, but the details will be omitted here.

Input cards for this run are shown in Appendix A which also contains the output from this sample case. The density DEN is not entered on the first input card for each of the first two Basic Parts, so the default value DEN1=.2750 will be used, and the number of sectors NJ is not entered on any cards so the default number NJ1=1 is used always. The 1 in column 20 of the Option Card directs the program to list only the first radius at each end of each Basic Part. In this case, the Basic Parts all have one complete sector so these two radii R1(1) and R2(1) are the only ones.

The properties of the Known Part entered on the last two input cards were obtained by inputting that Part by itself, identical to Part 2, but with arbitrary DZ. Because in that run the Part in question constituted the entire object, the overall center of gravity was also the Part CG and so the "COMPONENTS OF INERTIA TENSOR WITH RESPECT TO C.G." printouts on that output are the input quantities needed for the present run.

The effect of the upward shift in the core in this same case can be seen from the output, the overall center of gravity is in the xz-plane at $x=0.113$ instead of on the z-axis; the third principal axis has a positive x-component so it is tilted slightly upwards (by about 1/30 degree); and Ixz does not equal zero as it would if the core were symmetric about the z-axis.

1.5.2   Sample Case 2

The object to be analyzed in this sample case is shown in Figure 7 and the axes shown are those with respect to which the moments and products of inertia are required, that is, they are the Reference Axes. It is convenient to divide the object into three sections, the left one consisting of the half cylinder, the right one the rectangular parallelepiped, and the middle section the semi-wedge shaped object.

The left section is a Basic Part because it is a 180° sector of a right circular cylinder. Its Input System z-axis must lie along the axis of the cylinder (parallel to the Reference y-axis) and if the Input x and y-axes are chosen parallel to the Reference System z and x-axes

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respectively, then the Input System will be a right-handed
orthogonal coordinate system. The origin must be chosen
so that the previously chosen Input z-axis lies on the
axis of the cylinder, so its x and z-coordinates XP and
ZP must be 1.0 and 0.0. The y-coordinate YP is arbitrary
but will be chosen at 0.0 for simplicity. Then, when
considered as a Basic Part (NTYPE=1) described with respect
to the Input System above, the Part labelled PART 1-RP
has length 3.0, density equal to the default value, and
displacement zero, because YP=0. Because the angles are
measured from the x to y Input Axes, TH(1)=270° and
TH(2)=90°, and the radius at each end is 1.0. NR and NTR
are chosen as 1 to indicate that Rotation and Translation
information will follow, consisting of the directions of the
x, y and z Input System Axes (0,0,1), (1,0,0) and (0,1,0)
respectively (in Reference Coordinates) and the Reference
Coordinates (1.0, 0.0, 0.0) of the origin of the Input
System.

The middle section can be more easily analyzed by
considering the xz-cross section in which the dotted line
(actually a plane) has been drawn dividing the section
into a Concave Part and an Angular Wedge, which have been
labelled Part 2 and Part 3 in the figure.

For Part 2, the Input System Axes are chosen
parallel to those of Part 1, but with origin at (0.0,
0.0, 4.0), again in the xz-plane. The length is 3.0, the
density is the default value, and the displacement
DZ is zero. The angles must be measured from the Input
x-axis (parallel to DE) about point D counterclockwise
to the lower radius (line DA) and to the upper radius
(line DB). The lengths of these lower and upper radii
DA and DB are approximately 3.4164 and 3.1509 and the angles
EDA and EDB are approximately 155.147° and 169.687°
respectively. The exterior center for the Concave Part is
at the point F, which has Input System coordinates XC=-4.0
and YC=1.0, the Rotation information is as before, and
the Translation information reflects the Reference Coordinates
of the origin of the Input System of Part 2.

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The other region in the middle section, labelled Part 3, is an Angular Wedge. There are many ways to chose the Input System but it is convenient to choose the same system as for Part 2. Then the length is 3.0, DZ=0.0 the radii DC and DA have lengths 2.0 and 3.4164, and the angles EDC and EDA are 90.0° and 155.147° respectively. The Rotation/Translation information is the same as for Part 3 because the Input System is identical.

It should be realized that this middle section could be modelled as a single Standard Wedge (with xz-cross section ABCD) which would introduce a slight inaccuracy because segment AB is actually an arc of a circle rather than a straight line. This is a simple example of the decisions on accuracy versus input preparation time which have to be made in using MOMENTS-II. Here, the exact representation, seen by drawing line AD is preferred because it is also quite simple. The use of auxiliary lines in modelling odd-looking shapes into combinations of Angular Wedges and other Parts is an "art" which can play an important role in the use of MOMENTS-II.

Finally, the rightmost section is exactly a Standard Wedge. The requirement that the xz-Input plane divide it in half forces the choice of YP=1.5, and ZP has been chosen equal to 4.0 to allow DZ=0.0. (In fact, any combination of ZP and DZ such that ZP+DZ=4.0 would be valid here.) The Input Axes are parallel to the Reference Axes so that the length H (always measured parallel to the Input z-axis) is 5.0, the heights at both ends are 2.0 and the width W (measured parallel to the Input y-axis) is 3.0.

This completes the description of the modelling of the piece. The Input cards for this case are shown in Appendix B and the output follows.

Since it was convenient to rotate the axes for 3 out of the 4 Parts, it would have been equally simple to redefine the Reference Axes to be respectively parallel to the axes of this common Input System and to obtain the properties with respect to the new Reference Axes. Since in this case the rotations are all merely axis interchanges,

the desired results could have been easily read from the output. This was not done in order to better illustrate the use of Rotation information. It should be noted that this simple situation in which the Rotation information describes merely interchanges of axes is not the most general case. To obtain properties with respect to a set of axes not parallel to the first, the results with respect to the CG-Axes could be used in treating it as a single Known Part to find the properties in a second run, or one could change all the Rotation information instead, which (as mentioned in 1.1.3) is the less desirable method.

1.5.3 Sample Case 3

The purpose of this sample case is to outline the method for analyzing an object, working from its blueprints. It utilizes the technique of dividing the object into sections, which can more precisely be called "Levels" (i.e., regions between parallel planes which are perpendicular to the axis of the object), in order to reduce the dimensionality of the problem. The method consists of choosing these levels so that the xy-cross-section is constant within any one Level, so that using one blueprint for each one can effectively reduce the problem to a number of two-dimensional ones in which only one Level is analyzed at a time. Thus, one models each Level individually and combines the separate inputs from them to describe the complete object to the MOMENTS-II program.

The present sample case illustrates the above technique by considering a single Level of a multi-Level object. Figures 8a and 8b are the two views (A and B) of the object and Figures 9a and 9b are its two section views (A-A and B-B); all four views are taken from the actual blueprint of the object. On the front view (A) the proper border for the Level under consideration is outlined darkly, and dotted lines have been added to model the Level into Parts. Since these are cross-sectional views, the resulting Parts will generally be Basic Parts (Floating Sectors) with equal radii at the ends (sectors of a circle, in two dimensions), Angular Wedges (triangles, in two dimensions) and Concave Parts. In the present case the dotted lines divide the Level into 11 Parts, not including the holes which are treated last as simply cylinders (Basic Parts, NTYPE=0) with negative densities. Letters for labelling points as well as numbers for identifying Parts have been added to view A to facilitate the descriptions.

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The Level under consideration here is the middle one of three in the actual object. Therefore the IDENTS for the Parts in it will be of the form "NO. N..L2," where N is the Part number (here 1<N<11) and L2 stands for Level 2, since in this case only the middle or second Level is considered.

The x and y-axes shown in view A are the Reference Axes and the z-axis points outward from the blueprint. Because the problem has been reduced to a two-dimensional one, these x and y coordinates alone will be used to describe points on the cross section; they should be assumed to be with respect to the system with origin at O unless otherwise specified.

The first Level, whose length is .094, was assumed to be positioned with its lower-z face on the xy-plane (at z=0) of the Reference System. The Input Systems for every Part in this case are chosen with their origins in the Reference xy-plane, so that the Translation information (where necessary) is always of the form (XP, YP, 0) and the z-displacement DZ for every Part in this second Level is set to .094. Also, the "length" of the second level is .095 which therefore serves as the value of H for every Part.

A description of the process as applied to this sample Level will now be presented; each of the Parts into which it has been modelled will be considered individually in numerical order corresponding to the numbers on view A. Although most of the information needed to compute the necessary angles and radii comes from view A, some has been taken from view B.

The first Part, AOB, is a Concave Part which will be identified as "No. 1...L2". R1(1) and R1(2) represent the lengths of OA and OB which can be found on the blueprints as .325 and .475 respectively. The angles are given by TH(1) = 355.0° and Th(2) = 19.800°, the latter being found by intersecting the two circles which are centered at S with radius 0.24 and at O with radius 0.475. The exterior center is at the point S whose coordinates are XC = .56285 and YC = -.049243 (i.e., .565 cos 355° and .565 sin 355°).

The second part, BOC, called "NO. 2...L2" is a
Basic Part (NTYPE=1) with TH(1)=19.800° and TH(2) =
137.860°. The latter can be seen by observing that point
P lies on the line through F making an 80° angle with the
negative x-axis and also on the circle with center at U
and radius .362. The coordinates of F can be read from
view A as (-.18,0.0) and those of U can be read from view
B as (-.202, -.168), so that the coordinates of P are
(-.214171, .193793) which yields TH(2). The radii are
given by R1(1) = R2(1) = 0.475 as can be seen from view
B. It should be noted that C is simply the point where
line OF meets the large arc and the dotted line was drawn
here because it delineates the largest Part 2 can be if one
requires point P to fall outside of this Part.

The third Part, CPD, will be approximated by a
Basic Part (NTYPE=1) with radii R1(1) = R2(1) = .19246 and
angles 137.86° and 164.414°. It is described with respect
to its Input System with origin at P (not at O) so that NTR=1
and (XP, YP, ZP) = (-.214171, .193793, 0.0); ZP is zero
because the origin of the Input System lies in the Reference
System xy-plane and DZ takes care of the z-displacement.
This is an approximation only, because CD is actually an
arc of the larger circle (radius = .475) centered at O
rather than of a circle centered at P. (Note that no
approximations whatsoever were involved in the previous
two sample cases, because all sections there could be
represented exactly as combinations of Parts, whereas here
this is not the case.) Calculating the coordinates of
D, the intersection of line DE and the large circle centered
at O, one obtains (-.405610, .247195). (The line DE is
considered as shown in view B as parallel to the line y=
(tan 114°)x and .27 units from it.) The angle, measured
about P as center, of CP is the same as its angle about
O because CPO is a single straight line. The upper angle
(about P) of the line DP can now also be found knowing the
coordinates of both D and P, and this is also as given
above. Computing the lengths of both CP and DP, they
are (expectedly) unequal, but their values .18617 and .19875
are close enough so that the approximation of CPD as a
Basic Part (sector of a cylinder) centered at P will be
accurate enough if one uses as its radius .19246, which is
the average of the two.

Since arc CD is actually on a circle centered at O, one could alternately allow Part 2 to extend to D (rather than to C), having line OD (not drawn) as its upper boundary. This would remove the necessity of the Part 3 approximation to CPD described above but would require (the deletion of) Angular Wedge DPO as Part 3. Thus, it would be more accurate, require the same number of Parts, and not affect any of the other Parts, so that it is superior to the method used above. This illustrates the type of modelling decisions which must be made in using MOMENTS-II. The degree of accuracy required and the time available are the basic criteria for making these decisions. In the present case, a comparison run using the exact representation shows that the loss in accuracy due to the model actually used is small.

Part number 4 (EDP) will be approximated by a Concave Part about D as center with radii $R1(1)=DE=.10052$ and $R1(2)=DP = .19878$ and angles $TH(1) = 294.0°$ and $TH(2) = 344.414°$. Because the origin of its Input System is at D, the Translation information $(XP, YP, ZP) = (-.40561, .247195, 0.0)$ must be included for this Part. The exterior center is at the point U given by $(-.202, -.168)$ with respect to the Reference System, so that the inputs (XC, YC) with respect to the Input System center at D are given by $(.203610, -.415195)$. The upper radius DP has already been calculated and the upper angle of this Part can be calculated from the upper angle of Part 3.

EDP was considered a Concave Part because EP is a circular arc; that is only an approximation because DE is not a perfectly straight line but has rounded ends. Since the arc is relatively short (in angular measure) it is nearly a straight line and EDP could have been modelled as an Angular Wedge. This approximation is in fact a good one (as a comparison run would show) and could have been used with only a negligible loss in accuracy.

Part 5 (POF) is an Angular Wedge centered at O. Its radii and angles are easily seen to be $RR1 = OP = .288834$, $RR2 = OF = .18$, $TH(1) = 137.86°$ and $TH(2) = 180.0$.

Part 6 (FOG) is also an Angular Wedge centered
at O. Its radii are RR1 = OF = .18 and RR2 = OG = .352463
and its angles are given by TH(1) = 180.0° and TH(2) =
231.621°. The lower quantities are obvious and the upper
ones can be verified by computing the coordinates of
the point G, the intersection of line FG and the circle
about Q with radius .195. Since the coordinates of Q
are (-.031314, -.222810), which are obtained from its
polar coordinates (r, θ) = (.225, 262°), the resulting
coordinates of G are (-.218832, -.276302), from which the
upper quantities given above are found.

Part 7 (GOQ) is another Angular Wedge centered at
O. Its radii are RR1 = OG = .352463 and RR2 = OQ = .225
and the corresponding angles are TH(1) = 231.621° and
TH(2) = 262.0°. All of these quantities have either already
been computed or can be read from view A.

Part 8 (GQH) is a Basic Part (NTYPE=1), described
about Q as center, whose coordinates are known. Since the
coordinates of G are also known, the lower angle TH(1)
of this Part is easily calculated. To calculate the upper
angle TH(2), one applies the law of cosines to isosceles
triangle GQH (not drawn) to obtain angle GQH = 141.327°.
Thus TH(1) = 195.922°, TH(2) = 337.249°, and R1(1) =
R2(1) = .195.

Part 9 (HQI) is an Angular Wedge described about
Q. Its lower quantities are RR1 = .195 and TH(1) = 337.249°
and one requires the coordinates of I to find the upper
radius QI. The coordinates of H in the Input System with
origin at Q are (.179828, -.075412), and assuming I to be
at the intersection of the lines HI (parallel to FG and
.368 units to the right of T) and QJ, the coordinates of
I with respect to the Q-centered system are (.194714,
.030505). Thus the length of QI = RR2 = .197089 and the
upper angle TH(2) = 8.904°.

It should be noted that NTR = 1 for Parts 8 and 9
and the Translation information (XP, YP, ZP) = (-.031314,
-.222810, 0.0) must be supplied for both of these describing
the location of the point Q, which is the center of the
Input Systems for both of these Parts.

Part 10 (QOI) is an Angular Wedge described about O. Its lower radius and angle are RR1 = OQ = .225 and TH(1) = 262.0°. Its upper angle is found by noting that J lies on a circle with center O and radius .325 and has y-coordinate -.175. Thus the coordinates of J are (.273861, -.175), the upper angle TH(2) = 327.421° and the corresponding RR2 = OJ = .325. Part 10 is not included as a piece of Part 7 because GQI is not a straight line.

Part 11 is a Basic Part (NTYPE=1) with R1(1) = R2(1) = .325 and angles TH(1) = 327.421° and TH(2) = 355.0°.

The final task is to delete the four holes centered at O, Q, R and T. The radii of these (cylindrical) holes, which are treated as Basic Parts (NTYPE=0), can be read from two section drawings A-A and B-B (Figures 9a and 9b) and the locations of their centers, which are used as origins of their Input Systems, can be obtained from the basic blueprints.

For Hole 1 centered at O, the density is -DEN, R1(1)=R2(1)=.0655 and no Translation information is necessary.

For Hole 2 centered at R, the density is - DEN, R1(1) = R2(1) = .0466 and the Translation information locating the origin of the Input System at the projection of the point R on the xy-plane is given by (XP, YP, ZP) = (-.05, .29, 0.0).

For Hole 3 centered at T, R1(1) = R2(1) = .0466 and the Translation information is given by (XP, YP, ZP) = (.212, -.095, 0.0).

For Hole 4 centered at Q, R1(1) = R2(1) = .076 and for this Part (XP, YP, ZP) = (-.031314, -.222810, 0.0).

All holes are accounted for (deleted and filled with other material if necessary) at the very end, so that if a hole penetrates through more than one Level, it is treated only once - not for each Level it

crosses, by setting its length H equal to the sum of the
lengths of the Levels it crosses (as long as its radius
is unchanged). In the present instance, the holes are
deleted after only one Level because the overall object
being considered here consists of only one Level. Thus
the holes are all Basic Parts (NTYPE=0) with DZ = .094
and H=.095 as for all the other Parts in this Level 2
of the overall object. Throughout, the density DEN
of the (brass) Parts will be used in all cases, except for
the holes, all of which are assigned a density of DEN = -.308.
Note that the units used in this Sample Case are consistent
because the length dimensions are all given in inches. The
inputs and output for this Sample Case are listed in
Appendix C.

1.5.4 Sample Case 4

The purpose of this sample case is to illustrate
the inputs and outputs for all 5 types of Basic Parts.
As will be pointed out, certain of these are not expressed
using the simplest means possible. This is done to illus-
trate the longer printouts (for NPRNT=2) and when it is,
it will be pointed out along with the simpler method.

The first of these Basic Parts, for which NTYPE=0,
consists of 24 equiangular sectors of cylinders (each pair
of corresponding lower-z and upper-z radii are equal),
all with the common density 0.2, length 1.0, and lower-z
coordinate (displacement) 4.0. Because of the arrange-
ment of the radii in three groups of eight, the same Part
could have been (more easily) represented by a Basic Part
(NTYPE=0) with three sectors and radii of 1.0, 2.0, and
3.0; the present form is used for illustrative purposes
only.

Considering the Rotation/Translation information,
obviously if the directions and origin of the Input System
for this Part are as indicated, then this system must be
identical to the Reference System. In fact, this is true
for all five Parts in this sample case, and although it
could have been indicated each time by omitting all Rotation
and Translation information, "dummy" data has been included
here also to obtain a printout containing all inputs for Basic
Parts.

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The second Basic Part, NTYPE=1, is a Floating 90°
Sector, from 45° to 135°, with density 0.1, length 1.0,
displacement 1.0, with radii 1.0 and 2.0 (at the z=1.0 and
z=2.0 ends respectively). Again the Input System of this
Part has been chosen to be identical to the Reference
System.

The third Basic Part, NTYPE=2, consists of the
remaining three quarters of the frustum from which the
second Basic Part was taken. Since the lower angle of
this Part is assumed to be at 0°, it consists of three
sectors, from 0° to 45°, 45° to 135° and 135° to 360°.
The first and last of these have been given end radii of
1.0 and 2.0 as for the previous Part, but the second has
been omitted by setting both of its end radii equal to
zero (which is the only proper way to omit a sector of
a Basic Part). Thus, the second and third Basic Parts
together could more easily be represented as one complete
frustum with radii 1.0 and 2.0, length 1.0 and density
0.1, situated about the Reference System z-axis from z=
1.0 to z=2.0.

The fourth Basic Part, NTYPE=3, consists of 24
equiangular sectors (15° each) whose density RH(J),
J=1,...,24 varies by sector. Because of the arrangement
of the radii and density, this same Part could have been
represented by a Basic Part (NTYPE=3) with only three
sectors with radii (at both ends) of 1.0, 2.0 and 3.0
and densities of 0.1, 0.1 and 0.2 respectively. One should
note that the density DEN for this Part is inputted as
0.0 on Card type 3a, so that the value DEN1 = 0.1 is
substituted for it and the result is as if DEN = 0.1
had been read in initially.

Also, the first 8 values of RH(J) are blank; these
will be read in as zeroes and the default value DEN =
0.1 (itself a default value) for this Part will be used
in its place.

The final Basic Part, NTYPE=4, consists of 24
sectors with varying angular measure and density. Again
because of the arrangement of radii, upper angles and
densities, this Part could be more simply represented by
a Basic Part (NTYPE=4) with three sectors, from 0° to
80°, 80° to 200°, and 200° to 360°, with radii 1.0, 2.0,
and 3.0 (both ends) and densities 0.1, 0.1, and 0.2
respectively. Each of its sectors extends from z=0.0
(i.e., DZ) to z=1.0 (DZ+H).

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In all cases, only the (optional) data which applies as indicated by NTYPE, NJ, NR and NTR will be read in and printed out. If the input data does not match that signalled by these indicators, errors will invariably occur. A printout with NPRNT=2 will show all the information which has been supplied by the user (generally in the same order as on the input cards) and is therefore of great use in checking inputs. The inputs and output for this Sample Case are listed in Appendix D.

2.0 PROGRAMMER'S GUIDE

The computer program MOMENTS-II is written in FORTRAN (EXTENDED) for CDC 6000 series computers, and its purpose is to calculate the moments and products of inertia and other physical properties of inputted objects. MOMENTS-II is not highly machine dependent and therefore can easily be converted to run on other computers. It is sufficiently modularized to permit easy understanding and modification.

This chapter provides sufficient information about the actual FORTRAN routines to enable one to follow the calculations in MOMENTS-II and to modify the coding if desired. It explains the main routine and the subroutines, providing flowcharts and dictionaries of FORTRAN variables (where necessary) in addition to explaining the purpose of each routine and the calculations performed, with references to the User's Guide and to the Analyst's Guide where necessary.

2.1 MAIN ROUTINE

This section describes the main routine, which controls the flow of the program including reading the inputs, calling the subroutines to do the calculations and writing the output. All I/O statements appear in the main routine and all major computations are performed in the subroutines, although certain minor calculations are performed in the main routine. The following subsections further describe the main routine, including an explanation of its execution, a dictionary of its FORTRAN variables and a flow chart of its logical steps.

2.1.1 Execution of the Main Routine

The organization of the Main routine is quite straight-forward; aside from reading all the inputs and writing all the outputs, the basic functions it performs are calling the proper subroutines to calculate the properties of the inputted Parts (one at a time), and using these to calculate the desired properties of the entire composite body. After initializing certain indices, linecounts and properties, MOMENTS-II first reads the Title and Option Cards to find the default values, the type of printout required, and the number of each type of Part for the current case. A summary of this information is then printed out after which the routine begins its first task of analyzing the Parts individually.

The following sequence of logical steps is then performed for each "Unknown Part" until only Known Parts remain to be read in. All the input cards pertaining to the Part are read in (the program "knows" what type of Part is being read in from the Option Card and the assumed order of the input cards) and the inputs are printed according to the option NPRNT. Next, the main routine calls either SUBROUTINE BASIC, WEDGE, WEDGE1 or CONCAVE, depending on whether the Part is a Basic Part, Standard Wedge, Angular Wedge or Concave Part respectively. These routines, as explained in the sequel, always calculate the relevant properties of the Part with respect to its Input System. If Rotation information is supplied for the Part (NR=1), SUBROUTINE ROTATE will be called to use these properties to calculate the properties of the Part with respect to a set of axes with origin coinciding with that of the Input System but parallel to the Reference Axes. If Translation information is supplied (NTR=1), SUBROUTINE TRANSL will be called to use this information to calculate the properties of the Part (usually) with respect to the Reference System. The output for a Part, containing its mass, volume (both negative if the Part is to be deleted) and center of gravity (in Reference Coordinates) is then printed out. (The moments and products with respect to the Reference System are not printed here, but this capability could be easily added.) The program then adds these properties (moments, products, mass, volume) to the current totals which have been initialized to zero.

After this complete process has been performed for all Basic Parts, all Wedges and for all Concave Parts, the program must next consider the Known Parts. This is basically the same as in the previous cases except that there are no calls to any subroutine to calculate the properties (with respect to its Input System) because they are already known. The procedure in this case is as follows: After reading the inputs and writing them, the Reference Coordinates of the center of gravity of the Known Part (CGX, CGY, CGZ) are saved in XP, YP, ZP because they are also the coordinates of the origin of its Input System, and if necessary, ROTATE is called to adjust the (known, inputted) moments and products with respect to this Input System, to a coordinate system with origin at the center of gravity of the Part but with axes parallel to the Reference Axes. The difference is that here SUBROUTINE TRANSL is called automatically to translate the properties from the center of gravity based system to the Reference System. Because subroutines ROTATE and TRANSL require coordinates of the center of gravity in

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the same system as the moments and products (the Input System) at the time, the center of gravity coordinates CGX, CGY, CGZ of the Known Part in the present (CG) system are here set to zero before calling either ROTATE or TRANSL. As before, these properties are then added to the current sum.

When all Parts have been processed, the total (additive) properties have been computed (with respect to the Reference System) by merely accumulating the individual results. It should be remembered that Parts to be deleted, signified by either negative DEN (for Unknown Parts) or negative AMASS (for Known Parts), will have the signs reversed on their moments, products, mass and volume, i.e., the mass and volume of objects to be deleted will always be negative. Thus for these objects, the accumulation of their properties will actually be a subtraction (addition of the negative), as is proper when deleting a Part.

The calculation of the overall center of gravity and net volume is then performed, after which the moments and products are transformed in the main routine, via the same type Moment and Product translations as in TRANSL and using equations (43) and (45) specialized to the case $\bar{x}=\bar{y}=\bar{z}=0$ (see next to last paragraph in 2.3.2) to the CG-System of the overall object. Then SUBROUTINE EIGENV is called to find the eigenvalues and eigenvectors of the inertia tensor (the symmetric matrix whose six upper triangular components are the moments and products of inertia - see 2.6) which are called the principal moments and principal axes of the entire body. All of the above quantities are printed on the final page of output.

2.1.2 Major FORTRAN Variables in Main Routine (floating point)

VARIABLE	EXPLANATION (Corresponding Symbol in Ch. 3)
A(6)	Upper half of symmetric inertia tensor matrix $$\overset{CG}{I_{xx}}, \overset{CG}{I_{xy}}, \overset{CG}{I_{yy}}, \overset{CG}{I_{xz}}, \overset{CG}{I_{yz}}, \overset{CG}{I_{zz}}$$
AMASS	Mass of a Part (m)
CGX,CGY,CGZ	Coordinates of CG of a Part $(\bar{x}, \bar{y}, \bar{z})$
DEN	Density of a Part (p)
DEN1	Default density of Part
DZ	Minimum (Input System) z-coordinate of Part (D)
E	Eigenvectors or principal axes of inertia tensor matrix
H	Length of a Part (H)
RH(72)	Array of densities (by sector) for a Basic Part
RM(3,3)	Rotation matrix (Rij)
RR1,RR2	Heights or "radii" of Wedges (R_1, R_2)
RSQ	Sum of squares of the CG coordinates $(x^2 + y^2 + z^2)$
R1(72)	Lower z-coordinate end radii of Basic Parts (R_1)
R2(72)	Upper z-coordinate end radii of Basic Parts (R_2)
"S" symbols	These symbols, all beginning with the letter S, represent properties of the entire object rather than of a single Part. The symbol remaining after deleting the prefix "S" indicates what the symbol stands for and can generally be found in this table. For example, since CGX is the symbol for the x-coordinate of the CG of the Part, SCGX is the symbol for the x-coordinate of the CG of the entire object.

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TH(72)	Upper angles of sectors of Basic Parts (t,T)	
VOL	Volume of a Part (V)	
VOLN	Negative volume, corresponds to negative density.	
VOLNT	VOLNT = SVOL + SVOLN = net volume of object	
W	Width of Standard Wedge (W)	
XC, YC	Out-center of cylinder for defining Concave Part.	
XP, YP, ZP	Reference Coordinates for origin of Input System of a Part.	
X2,Y2,Z2	SXCG*SXCG, SYCG*SYCG, SZCG*SZCG $(\overline{X}^2, \overline{Y}^2, \overline{Z}^2)$	

All other X, Y, and Z symbols are defined by integrals:

X	$\iiint x\,dm$	(Myz)
Y	$\iiint y\,dm$	(Mxz)
Z	$\iiint z\,dm$	(Mxy)
XX	$\iiint (y^2+z^2)\,dm$	(Ixx)
YY	$\iiint (x^2+z^2)\,dm$	(Iyy)
ZZ	$\iiint (x^2+y^2)\,dm$	(Izz)
XY	$-\iiint xy\,dm$	(Ixy)
XZ	$-\iiint xz\,dm$	(Ixz)
YZ	$-\iiint yz\,dm$	(Iyz)

2.1.3 Major FORTRAN Variables in Main Routine (fixed point)

VARIABLE	EXPLANATION
I	Index used for Parts
IDENT	Ten character (A10 format) identification for Parts
I1, I2, I3, I4	Number of Basic, Wedge, Concave and Known Parts (respectively)
J	Index used for sectors (J=1, NJ)
LN1, LN2, LN3 LN4	Line counts for output for Basic, Wedge, Concave and Known Parts
L1	Line increment for Basic Parts
NJ	Number of sectors in a Basic Part
NJ1	Default number of sectors in a Basic Part
NPART	Indicates type of current Part
NPRNT	Input controlling printing options
NR	Indicator for Rotation
NTR	Indicator for Translation
NTYPE	Indicator for kind of Basic Part
N1, N2, N3, N4	Number of Basic, Wedge, Concave and Known Parts (respectively)

2.1.4 Flow Chart of Main Routine

The following page contains a flow chart illustrating the logical steps in the main routine, corresponding to the outline in 2.1.1. Those operations which appear in parentheses in the flow chart are optional and depend on values of NPRNT, NR, and NTR for their inclusion or exclusion. The decision on whether there are more cases or not is simply a check on the end of the Input File, and the decision on whether there are more of a specific type of Part remaining is simply the result of a comparison of the number of the current Part (I1, I2, I3 or I4) with the total number of that type Part (N1, N2, N3 or N4, respectively).

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```
( 1 )  ┌─────────────────┐         ╱ More ╲   No      ( E N D )
       │ Initializations │ ──────▷ ╲ Cases? ╱ ──────▷
       └─────────────────┘          ╲    ╱
                                      Yes
                                       │
                                       ▽
                          ┌───────────────────────────┐
                          │  Read TITLE, OPTION Cards  │
                          │     Write Option Page      │
                          └───────────────────────────┘
```

```
     ( 2 ) ········▷ ( 30 ) ········▷ ( 50 ) ········▷ ( 60 )

     ╱ More ╲  No    ╱ More ╲  No    ╱ More  ╲  No    ╱ More  ╲  No
     ╲BASICS?╱ ───▷ ╲WEDGES?╱ ───▷ ╲CONCAVES?╱ ───▷ ╲KNOWNS?╱ ───▷
      ╲    ╱          ╲    ╱          ╲     ╱          ╲    ╱
       Yes             Yes             Yes             Yes
        │               │               │               │
        ▽               ▽               ▽               ▽
  ┌───────────┐  ┌───────────┐  ┌───────────┐  ┌───────────┐
  │ NPART = 1 │  │ NPART = 2 │  │ NPART = 3 │  │ NPART = 4 │
  │Read Inputs│  │Read Inputs│  │Read Inputs│  │Read Inputs│
  │(Write     │  │(Write     │  │(Write     │  │(Write     │
  │ Inputs)   │  │ Inputs)   │  │ Inputs)   │  │ Inputs)   │
  │CALL BASIC │  │CALL       │  │CALL       │  │(CALL      │
  │           │  │ WEDGE,1   │  │ CONCAVE   │  │ ROTATE)   │
  │(CALL      │  │(CALL      │  │(CALL      │  │CALL TRANSL│
  │ ROTATE)   │  │ ROTATE)   │  │ ROTATE)   │  │           │
  │(CALL      │  │(CALL      │  │(CALL      │  │(Write     │
  │ TRANSL)   │  │ TRANSL)   │  │ TRANSL)   │  │ Output    │
  │(Write     │  │(Write     │  │(Write     │  │ on FILE 1)│
  │ Output    │  │ Output    │  │ Output    │  │           │
  │ on FILE 1)│  │ on FILE 1)│  │ on FILE 1)│  │           │
  └───────────┘  └───────────┘  └───────────┘  └───────────┘
```

```
                  ┌──────────────────────────────┐
                  │ Accumulate Additive Properties│
                  │ with respect to REFERENCE AXES│
                  └──────────────────────────────┘
                               │
                               ▽
                  ┌──────────────────────────────┐
                  │  GO TO (2,30,50,60),NPART     │
                  └──────────────────────────────┘
```

```
  ( GO TO )  ◁── ┌────────────────────────────────────────┐
  (   1   )      │ Compute C.G. and Transform Inertia      │
                 │      Tensor to C.G.-SYSTEM              │
                 │ CALL EIGENV for Principal Moments, Axes │
                 │ Write Results on final Output Page      │
                 └────────────────────────────────────────┘
```

2.2 SUBROUTINE BASIC

This subroutine calculates the moments and products X, Y, Z, XX, YY, ZZ, XY, XZ and YZ and the properties AMASS, VOL, CGX, CGY, CGZ for a single Basic Part, using the necessary Dimensional inputs for the current Basic Part obtained from COMMON. After initializing the necessary properties to zero, the subroutine sets up the elements of the array of angles T(J), J=1,...NJ+1 defining the NJ (angular) sectors of the Part, in a manner depending on the type of Basic Part under consideration (as indicated by the value of NTYPE, see 1.3.2). For a Floating Sector (NTYPE=1) this merely amounts to converting the lower and upper angles TH(1) and TH(2) in degrees, to T(1) and T(2) in radians. In all other cases, T(1) is set to zero and T(J) for J=2,...,NJ+1 are set as follows. If NTYPE=0 or NTYPE=3, the angular measure DT of a sector is fixed (it equals $2\pi/NJ$), the angles are defined by T(J) = (J-1) * DT for J=2,...,NJ and T(NJ+1) = 0. If NTYPE=2 or NTYPE=4, DT is variable and the sector angles T(J) are computed from the upper angles TH(J) using T(J+1)=TH(J) * CONV, for J=1, ...,NJ. Note that whenever an upper angle TH(NJ) equals 360°, T(NJ+1) is set to zero to prevent unnecessary roundoff errors which would be introduced if one converted 360° to radians before taking trigonometric functions of the angle.

The subroutine next enters a loop which calculates the quantities for the sectors of the Basic Part one at a time, and then sums them up. This calculation produces the moments without the multiplicative constants, which are put in afterwards. If NTYPE=0 or NTYPE=3, DT is calculated in the loop for each sector as DT=T(J+1)-T(J). Because T(NJ+1) may have been set to zero, DT may be non-positive in which case 2π is added to it. If variable density is chosen (NTYPE=3 or NTYPE=4), the density DEN used in the loop for each sector is set to RH(J); otherwise, the common value DEN is assumed.

The moments XSQ, YSQ AND ZSQ are converted to the more useful XX, YY and ZZ in the final section, in which the constants are inserted. The coordinates of the center of gravity of the Basic Part are computed here in the form X/AM, where AM=AMASS unless AMASS=0, in which case AM=1, so that the coordinates of the center of gravity will be calculated as zero (because X=0) in the special case. This method is used in WEDGE, WEDGE1, and CONCAVE also. The calculations performed in BASIC follow equations (8) to (20) in 3.1.

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Most of the FORTRAN variables appearing in BASIC
are listed in 2.1.2 and 2.1.3. Those not listed there
may be found below:

VARIABLE (dimension)	EXPLANATION (symbol used in Ch. 3)
B,B1,...,B9	Temporary storage locations
CONV	Conversion factor $= \pi/180$
C1,...,C4	Constants
DT	Angular measure of a sector $=$ $T(J+1)-T(J)$. $(T-t)$
ST	$T(J+1)+T(J)$. $(T+t)$
T(73)	Angles defining the sectors, in radians (T's and t's)
XSQ,YSQ,ZSQ	$\iiint x^2 dm, \iiint y^2 dm, \iiint z^2 dm$ (Ix^2, Iy^2, Iz^2)

2.3 SUBROUTINES ROTATE AND TRANSL

The two subroutines ROTATE and TRANSL serve to transform the properties of Parts from initial coordinate systems to rotated and translated systems respectively. The basic application of these routines is to enable one to transform the moments and products of inertia, known with respect to the Input System of a Part, to the Reference System. The following two sub-sections will treat them individually.

2.3.1 SUBROUTINE ROTATE (RM)

The purpose of this routine is to transform the center of gravity and moments and products of inertia of any Part, already known with respect to some initial coordinate system, into these same properties but with respect to a rotated coordinate system, whose origin is, however, unchanged from the first. The relationship between these two systems is contained in the orthonormal rotation matrix RM (eqn. 48b) whose columns express the directions of the three axes of the initial coordinate system in terms of the rotated system's coordinates. Once the nine components of RM are known, ROTATE first calculates the new coordinates of the center of gravity by multiplying the old coordinates by RM (eqn. 55), and then it calculates the components of the inertia tensor with respect to the rotated system (eqn. 60) from the known properties of the Part with respect to the initial coordinate system (which it obtains from COMMON).

The most common application of this routine is to transform properties of the Part, known with respect to its Input System, into properties with respect to a coordinate system with origin coinciding with that of the Input System but rotated so as to be parallel to the Reference System. ROTATE performs this task when it is called by the main routine.

The other use of ROTATE is to transform the properties of a Wedge (created by the program) from its input coordinate system to a rotated system which is parallel to the Input System specified by the user in describing the Angular Wedge under consideration at the time. ROTATE serves this function when it is called by WEDGE1. A further explanation of this usage, which is also a specific case of its general purpose of

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performing a change of coordinates which is merely a
rotation of axes, is provided in 2.4 in the discussion of
WEDGE1.

It should be pointed out that there may seem to be a
certain redundancy in this routine because it transforms not
only the first moments but also the center of gravity (the
invariant mass relates the two sets of quantities according
to equation (13), in any coordinate system) instead of merely
transforming all the moments (including the first moments).
This is true to some extent, but the center of gravity trans-
formation has been included in ROTATE to avoid the necessity
of performing this calculation after each call to ROTATE.
Because of the inclusion of this (CG) computation here, no
equations for directly transforming the first moments are
necessary at all (i.e., no redundant calculations are
performed); instead, the first moments of inertia in the
rotated system are found from equation (13) in terms of the
mass and the (new) coordinates of the center of gravity,
which then have already been calculated. For this reason,
before calling ROTATE, the center of gravity must always be
expressed in the same coordinate system with respect to which
the inertia tensor is expressed.

In SUBROUTINE ROTATE, RM is the rotation matrix, RT
is its transpose, U is the inertia tensor, and X1, Y1, Z1
and TT are temporary storage locations. All other variables
are as in the main routine.

2.3.2 SUBROUTINE TRANSL (XP, YP, ZP)

The purpose of this routine is to transform the center
of gravity and moments and products of inertia of any Part,
already known with respect to some initial coordinate system,
into these same properties but with respect to a translated
coordinate system, which is, however, parallel to the first.
The relationship between these two coordinate systems
is contained in the "translation vector" whose components
XP, YP and ZP are simply the (new) coordinates of the origin
of the initial coordinate system. Once these three components
are known, TRANSL first calculates the new coordinates of the
center of gravity of the Part using equation (41) and then it
calculates the components of the inertia tensor with respect
to the translated system using equations of the forms (43)
and (45) from the known properties of the Part with respect

to the initial coordinate system (which it obtains from COMMON).
The same remarks as were made in the discussion of ROTATE
regarding the seeming redundancy in transforming CGX, CGY,
CGZ as well as X, Y, and Z apply here as well. It should be
noted that the computation of the transformed first moments
must be done after transforming the center of gravity because
the formulas used (43) and (45) are in terms of the new
coordinates of the center of gravity. If the center of gravity
computation were at the end of this routine instead of at
the beginning, then the correct formulas to be used in
translating the inertia tensor would be (42) and (44).

The most common application of this routine is to
transform properties of a Part, known with respect to a
coordinate system with center coinciding with the origin of
the Input System and parallel to the Reference Axes, into
properties with respect to the Reference System. TRANSL
performs this function when it is called by the main routine.

Another application of this routine is to translate
properties of Angular Wedges or Basic Parts (created by the
program) from some initial coordinate system to translated
systems which may coincide with the Input System specified
by the user in describing the (complete) Angular Wedge or
Concave Part. TRANSL performs this function when it is
called by either WEDGE1 or CONCAVE. This use is explained
further in 2.4 and 2.5 in the discussions of the routines
WEDGE1 and CONCAVE.

The final application of TRANSL is in converting from
the Input System of a Known Part to the Reference System.
In contrast to the situation for Basic, Wedge and Concave
Parts, TRANSL is always called in analyzing Known Parts
because the origin of the Input System for a Known Part is
always at the center of gravity of the Part, which generally
does not coincide with the origin of the Reference System.
(If the two do coincide, TRANSL is called anyway, but with
arguments equal to zero, so that it has no effect, except
possibly to change signs for deleted Parts as noted below
in discussing the use of F.) Therefore, in this case XP,
YP and ZP representing the Reference Coordinates for the
origin of the (Known) Part's Input System must be equal to
(the inputs) CGX, CGY, CGZ representing the Reference Coor-
dinates of the Part's center of gravity. As was the case for
ROTATE, the center of gravity must always be expressed in

the coordinate system with respect to which the inertia
tensor is expressed, before calling TRANSL. Therefore
CGX, CGY, CGZ are set to zero in the main routine since
obviously in its own Input System (CG-System) the coordinates
of the center of gravity of the Known Part are all zero.
Finally, if the mass of a Known Part is negative (i.e., if
the Part is to be deleted), F must be set to -1.0 in the main
routine (rather than the normal value of 1.0). Since AMASS
has already been made negative by the user, this has the
effect of reversing the signs of the inputted moments and
products, as they should be when a Part is to be deleted.
The volume VOL is reversed in sign in the main routine. Thus
most of the necessary sign reversals have been incorporated
in TRANSL (instead of in the main routine) which must be
called in this case anyway.

The reason that the main routine always calls ROTATE
before TRANSL (if both are needed) is that TRANSL requires
the (new) coordinates of the origin of the initial system.
These are available when the new system is the Reference
System, but if TRANSL were called first, its purpose would
be to translate from the Input System to another system
parallel to it, and a multiplication by a rotation matrix
would be necessary to find the coordinates of the origin of
the Input System with respect to this other coordinate system.
Therefore, calling ROTATE first, which already contains this
multiplication, avoids the necessity of intermixing rotational
and translational information and preserves the independence
of the two routines.

2.4 WEDGE SUBROUTINES

This section describes the two subroutines WEDGE and WEDGE1 whose purpose is to compute the properties of Wedges, Standard and Angular respectively, with respect to their Input Systems. Since an Angular Wedge can be viewed is a combination of two Standard Wedges, SUBROUTINE WEDGE1 calls SUBROUTINE WEDGE twice in calculating the properties of any Angular Wedge. The next subsections describe SUB-ROUTINE WEDGE, SUBROUTINE WEDGE1 and their interdependence, and provide a flow chart of WEDGE1.

2.4.1 SUBROUTINE WEDGE

This subroutine is capable of calculating the moments, products and other properties of a Standard Wedge with respect to its Input System, using the Dimensional inputs for the Wedge obtained from COMMON. The equations used in these computations are derived in 3.2.2 - 3.2.15.

SUBROUTINE WEDGE is called by the main routine to calculate the properties of Standard Wedges and also by WEDGE1 to supply the properties of the Standard Wedges making up every Angular Wedge. In the two cases WEDGE computes the properties with respect to the Input System specified by the user or by the program, respectively. The position of the Standard Wedge relative to its Input System is completely determined by DZ.

The variables $B1$ and $B2$ in this routine are set equal to the two end heights $R1(1)$. $R2(1)$ of the Wedge, and RD, RS and RP are used here as the difference, sum and product of $B2$ and $B1$. The variables A and B as well as all those beginning with the letter "R" are temporary storage locations, and all other variables in WEDGE have the same meanings there as in the main routine. None of the Dimensional quantities of the current Wedge are changed by this routine, i e., the values of all variables set before calling WEDGE are preserved.

2.4.2 SUBROUTINE WEDGE1

The purpose of this routine is to calculate the properties of an Angular Wedge with respect to its Input System. The method of accomplishing this is to represent

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each Angular Wedge as a combination (sum or difference) of
two Standard Wedges, each of whose properties are calculated
by calls to WEDGE. In essence, WEDGE1 does exactly what the
user would have to do in employing MOMENTS-II (but without
WEDGE1) to compute the properties of an Angular Wedge
with respect to its Input Axes; that is, the subroutine models
the Angular Wedge by describing to the program two Standard
Wedges which together comprise it, by providing the program
with their Dimensional and Positional information. (Since
this includes specifying their Input Systems, a convention
regarding the relation of each of these "sub-wedges" to
its Input System must be fixed, so that this information can
be uniquely assigned to these objects by WEDGE1 when they are
considered as Standard Wedges.) The properties of the
Angular Wedge (now viewed as a composite object) can then be
obtained by adding (or subtracting) together the properties
of its two component Parts.

The method upon which WEDGE1 is based depends on
the possibility of constructing any Angular Wedge from
either the sum or from the difference of two "component
Wedges" ("sub-wedges"); it varies significantly between
the two cases. The variations in logical steps between
the two cases are briefly summarized here; a more complete
explanation can be found in 3.5.

In the first case, WEDGE1 chooses a particular one
of these two sub-wedges and, once it sets up the Wedge's
Dimensional variables, it calls SUBROUTINE WEDGE1 to com-
pute the properties of this object with respect to its
Input System. It then calls SUBROUTINE ROTATE to trans-
form these to properties of the Standard Wedge with respect
to a set of axes parallel to the Input Axes of the Angular
Wedge (but with center at the origin of the Input System of
the Standard Wedge). Next, WEDGE1 sets up the Dimen-
sional variables for the other sub-wedge (eqn. 65), and
again calls WEDGE to compute the properties of this ob-
ject with respect to its Input System. In this first
case, the two sub-wedges have different Input Systems, which
do, however, have the same origin, so that ROTATE must be
called again (with a different rotation matrix) to obtain
the properties of this second Standard Wedge with respect
to the same system as for the first one. Then, the properties
of the combination of the two sub-wedges (i.e., the Angular
Wedge) are found with respect to this same set of axes; most
of its properties are simply the sum of those of its two

sub-wedges, except for the coordinates of its center of gravity, which are found by adding together the first moments and also the masses of the component Parts and using the standard formulae (13) on these sums. Finally, TRANSL is called to transform the properties of the Angular Wedge to its own Input System (as specified by the user), which is the desired result.

In the second case, where the Angular Wedge is represented as the difference of two Standard Wedges, these two "sub-wedges" are chosen to have the same Input Axes, so that WEDGE is called twice, to find the properties of these objects with respect to this common set of axes (once with positive density and once with negative density). These properties are summed as before to obtain the properties of the combination (the Angular Wedge) with respect to the Input System of the sub-wedges, and then these properties are rotated by calling ROTATE (once) to obtain them with respect to a coordinate system with origin coinciding with that of the common Input System but which is now parallel to the Input System of the Angular Wedge. Finally, as before, TRANSL is called to transform these properties to the Input System of the Angular Wedge, i.e., the one chosen by the user.

It should be noted that if the Input System and Reference System do not coincide, these results will again be sent to subroutines ROTATE/TRANSL (as controlled by the user's choice of NR and NTR) from the main routine (after leaving SUBROUTINE WEDGE1) as for other types of Parts, independent of the "TRANSLating" and "ROTATEing" that has already occurred internally unbeknownst to the user.

The specific information regarding the methods of deciding whether a sum or a difference of Standard Wedges should be used, choosing the two Standard Wedges which are combined to form the Angular Wedge, setting up the Dimensional information describing each Wedge before each call to WEDGE1, setting up the proper rotation matrices RM (containing the Positional information) before calling ROTATE, and setting up the arguments P1, P2 and P3 for the translation (before calling TRANSL), as well as the derivation of the equations used in these processes by WEDGE1, all can be found in 3.5.

2.4.3 WEDGE1 - Coding

The first section of coding is the initialization
section which contains the calculation of various lengths
and trigonometric functions of the angles in Fig. 12 which
apply to both cases, such as d, r, sin A, cos A and cos B
(from eqn. - 61, 62, 63) which are called D, BASE,
SINA, COSA and COSB respectively in WEDGE1. (The names of
the variables in WEDGE1 correspond very closely to the
notation in 3.5, which contains the relevant derivations.)
Included here are the unchanging elements in the rotation
matrix RM, which will be passed to ROTATE and used to
adjust the moments and products computed by WEDGE for
the two Standard Wedges (eqn. 69). Also, the original input
quantities (supplied by the user) DZ, H, R1(1), R1(2) (the
last two correspond to the user's RR1, RR2) are saved here
to be restored before returning to the main routine, since
they must be reset and passed to WEDGE in the course of
the calculations. For any of the sub-wedges passed to
WEDGE, their width W is set to the inputted length (H)
and since all the Wedges are true Wedges, R1(2) is set to
zero here as well (eqn. 65).

Next, the proper case is determined depending on
whether both angles A and B (see 3.5) are acute (NCASE=1)
or whether one of them is obtuse (NCASE=2). The path taken
in the remainder of the subroutine depends on NCASE and
follows the general procedure outlined in 2.4.2. The
major differences between these cases are the definition
of R1(1) for each sub-wedge, the number and elements of
the rotations performed. If NCASE=1, the properties of
the two sub-wedges are individually computed and rotated,
combined, and then translated. If NCASE=2, the properties
are individually computed, but the second Wedge (which is
actually contained in the first one) is given a negative
density so that the combining of properties which follows is
tantamount to a subtraction rather than to an addition.

It should be noted here that the initial center of
gravity calculations are always performed in WEDGE, but it
is the first moments of inertia (also calculated there)
that are summed before computing the center of gravity
using the usual formulas. In all cases, a correct set of
three center-of-gravity coordinates is stored in the variables
CGX, CGY and CGZ (in COMMON) before ROTATE or TRANSL are

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called. The rotation matrices for NCASE=1 are always fixed, but the one for NCASE=2 could match either of these two and depends on whether it is A or B which is obtuse (this is checked by the sign of COSB).

The final section of coding does not depend on NCASE and it is here that the call to TRANSL is made to adjust the properties of the (composite) Angular Wedge to its Input System. This call to TRANSL takes into account the DZ supplied by the user for the Angular Wedge, so that DZ is restored before this call; the other variables are restored before leaving the subroutine. All of the above is summarized by the flow chart to follow.
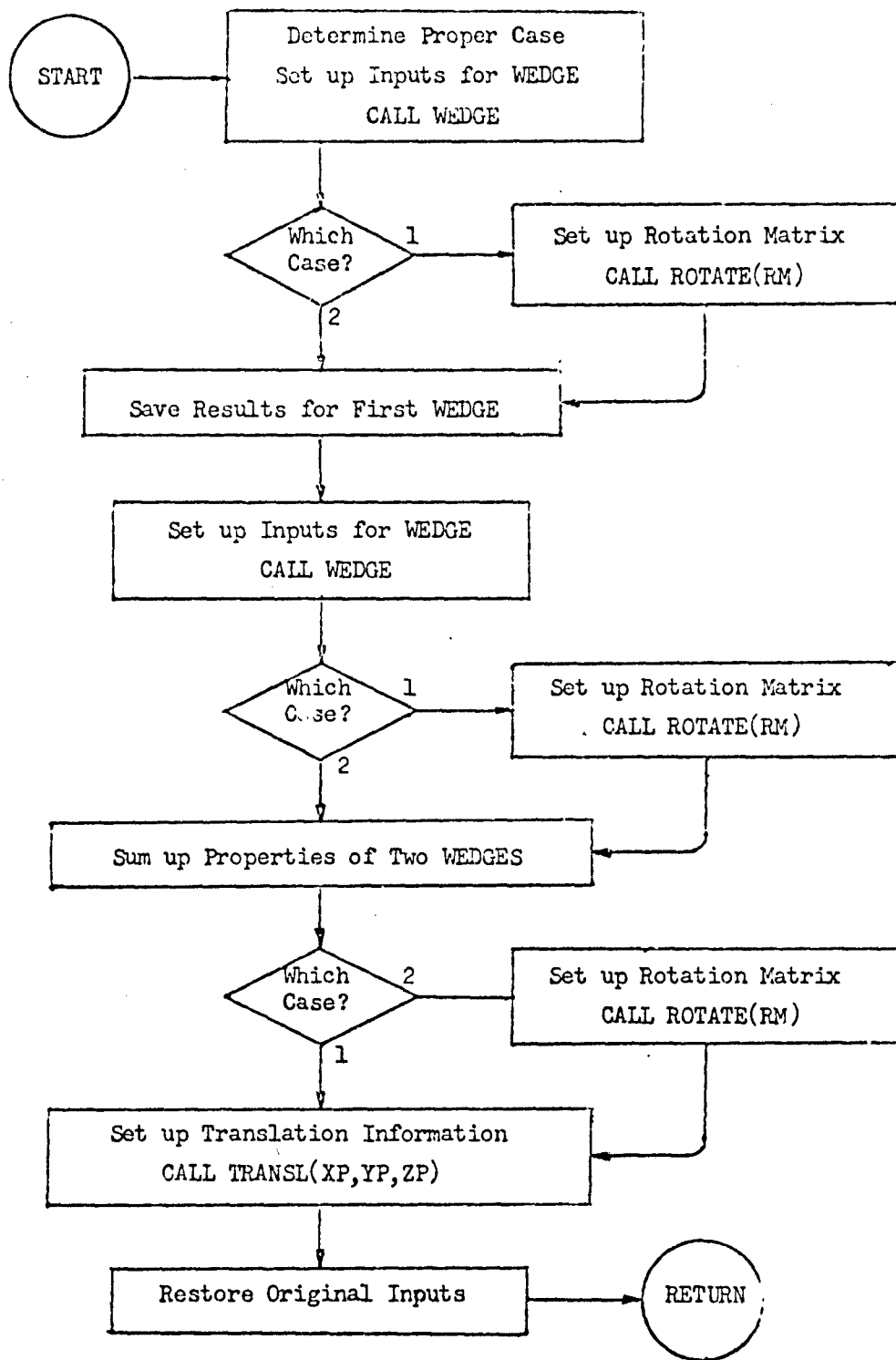
Flow Chart of SUBROUTINE WEDGE1

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2.5 SUBROUTINE CONCAVE (XP,YP)

The purpose of this routine is to compute the properties of Concave Parts, with respect to their Input Systems. The method of accomplishing this is related to that explained in the preceding section for Angular Wedges; that is, each Concave Part is modelled by the subroutine, by decomposing it into a combination of Parts which can already be handled by MOMENTS-II (without SUBROUTINE CONCAVE). Here, however, a Concave Part is constructed from three Parts - two Angular Wedges (which are added) and one Basic Part (Floating Sector, which is subtracted). As before, the properties of the resulting sum (the Basic Part is actually deleted; i.e., added with a minus sign preceding all its properties) are just the sum of the properties of the component parts, except that here, no rotations and only a single translation (independent of any specified by the user) are needed to find the properties of the Concave Part with respect to its Input Axes.

The next subsections contain a further explanation of the method (from a program point of view), a discussion of the coding of CONCAVE with reference to the relevant equations (derived in 3.6), and a flow chart of the subroutine.

2.5.1 Method of SUBROUTINE CONCAVE

The method upon which CONCAVE is based depends upon the possibility of constructing any Concave Part from a combination of two Angular Wedges and a Basic Part. As in 2.4.2 (for Angular Wedges), certain conventions for choosing the Input Systems of these three Component Parts must be fixed, and these will be obvious from the following explanation. One of the two Angular Wedges and also the Basic Part are assumed to have the same Input Axes. Their properties (with respect to their common Input System, whose axes are parallel to those in the Input System of the overall Concave Part, but whose center is at the point (XC, YC, DZ)) are computed by calls to the routines WEDGE1 and BASIC. These properties are added (actually, the properties of the Basic Part are subtracted from those of the Angular Wedge, by assigning the Basic Part a density equal to -DEN) to obtain the properties of the combination of the two with respect to the same axes.

A call to TRANSL with arguments XC, YC and DZ then trans-
forms these to properties with respect to the Input
System of the Concave Part. Finally, the properties of
the remaining "centered" Angular Wedge are calculated by
another call to WEDGE1 with respect to its Input System
(which coincides with the Input System of the Concave
Part), after which the properties are added as before.
The result is the desired properties of the Concave Part
with respect to its Input System.

The same two observations regarding computing the
center of gravity using the first moments of the compo-
nents, and the independence of any translation performed
here and any requested by the user (indicated by NTR and
called from the main routine) apply here as in 2.4.2.

Thus, the particular duties to be performed by
SUBROUTINE CONCAVE consist of calculating the dimen-
sions of the component Angular Wedges and Basic Part,
setting up the proper dimensions before each call to
WEDGE1 and BASIC, combining properties of the three
component Parts (by calling TRANSL and adding properties)
and restoring the original Dimensional inputs supplied by
the user, before leaving the routine. The method of con-
structing the Concave Part as a sum of three unique and
well defined objects rests on observing the diagram
(Fig. 5b) which illustrates the decomposition. The
equations representing the calculations performed by
Concave are derived in 3.6.

2.5.2 SUBROUTINE CONCAVE - Coding

The first section of SUBROUTINE CONCAVE saves
the inputs supplied by the user so that they will not
be destroyed when the variables in which they are stored
are used for calls to WEDGE1 and BASIC. The next section
sets up the inputs describing the Angular Wedge whose
center is at the "outcenter of the cylinder" (XC, YC, DZ),
following equations (70) to (73), and calls WEDGE1.
These results, including first moments in place of center
of gravity components, are saved. Next, the inputs for
the Basic Sector, also centered at the above mentioned
point, are set up and BASIC is called. Following this,
the properties are added and the center of gravity of the
combination is computed, after which TRANSL is called to
transform the properties to the Input System of the

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Concave Part. These properties are saved, the original inputs are restored (since they also describe the other Angular Wedge whose Input System coincides with that of the Concave Part), and WEDGE1 is again called to analyze this object. Finally, these properties are combined with the previous consolidation to obtain the desired results.

The variables with "1"'s appended to their FORTRAN names are used to store the results from WEDGE1 or from TRANSL, or to retain the initial inputs. The other variables are as in previous routines, except that those set up before the first call to WEDGE1 correspond closely to the notation in 3.6, in which their equations are derived.

```
        ┌──────────────────────┐
        │  Save Original Inputs │
(START)→─│     NPASS = 1        │
        └──────────────────────┘
                  │
                  ▼
        ┌──────────────────────┐
        │ Set up Inputs for WEDGE1 │
        │      CALL WEDGE1      │
        └──────────────────────┘
                  │
                  ▼
        ┌──────────────────────┐
        │  Save Current Results │
        │  (of WEDGE1 or TRANSL)│
        └──────────────────────┘
                  │
                  ▼
             ◇ Which ◇  ──2──▶  ┌──────────────────────┐
             ◇ Pass? ◇          │ Set up Inputs for WEDGE1 │
                  │1             │      CALL WEDGE1      │
                  ▼              └──────────────────────┘
        ┌──────────────────────┐        │
        │ Set up Inputs for BASIC │      │
        │      CALL BASIC      │        │
        └──────────────────────┘        │
                  │                      │
                  ▼                      │
        ┌──────────────────────┐        │
        │  Sum up Properties   │◀───────┘
        │  Compute C.G.        │
        └──────────────────────┘
                  │
                  ▼
             ◇ Which ◇  ──2──▶  (RETURN)
             ◇ Pass? ◇
                  │1
                  ▼
        ┌──────────────────────┐
        │  CALL TRANSL         │
        │  NPASS = 2           │
        └──────────────────────┘
```
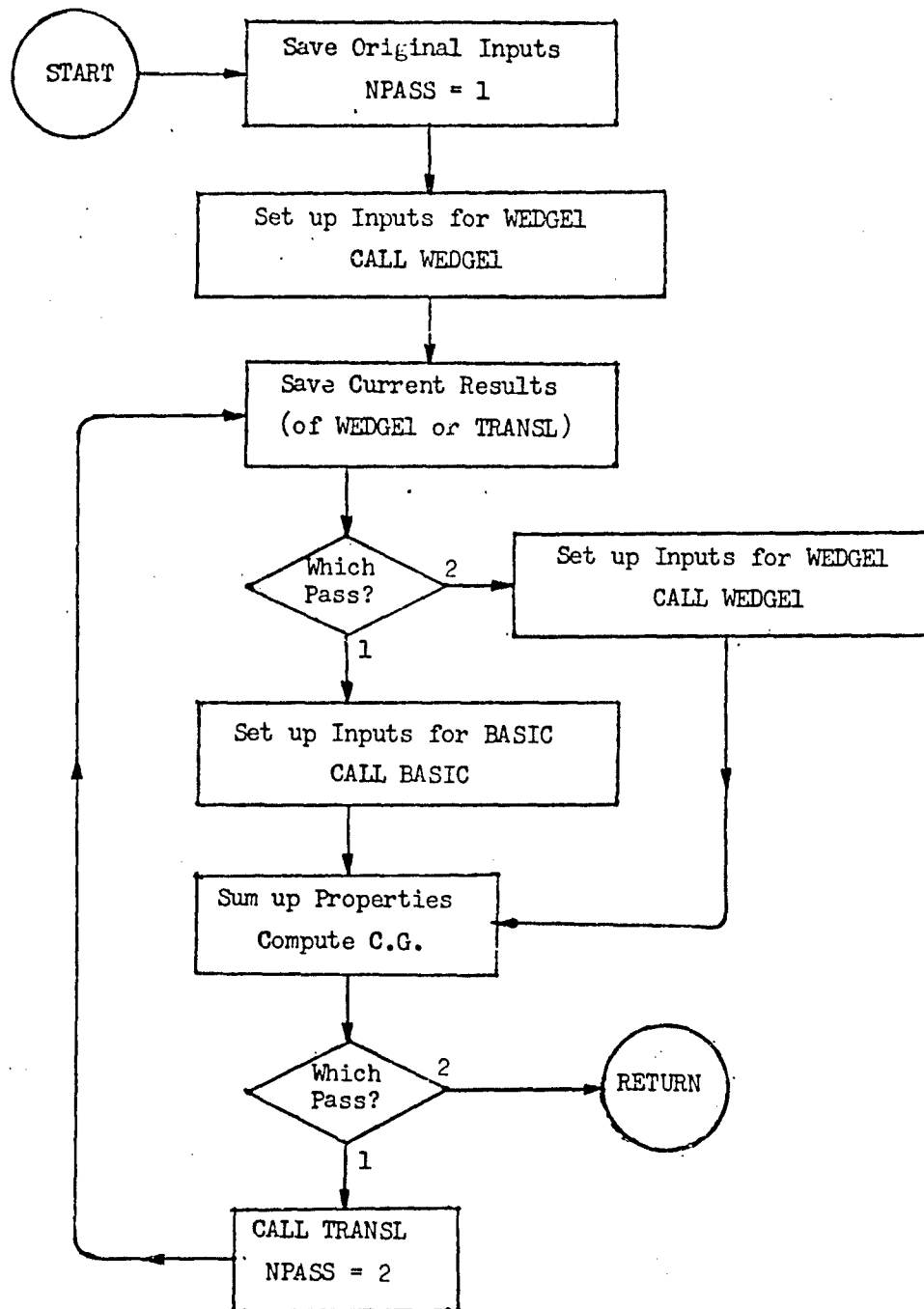
Flow Chart of SUBROUTINE CONCAVE

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2.6 SUBROUTINE EIGENV (A,E)

This routine calculates the eigenvalues and eigen-
vectors of the symmetric inertia tensor (stored in the
array A); the eigenvectors are the principal axes and
the eigenvalues are the principal moments of the overall
object. EIGENV uses a standard Jacobi iteration scheme,
as described in Ref. 4, for example, which employs a
sequence of simple rotation matrices to repeatedly trans-
form the matrix A until its off-diagonal elements are suf-
ficiently reduced in magnitude, at which time the symmetric
matrix has been nearly diagonalized and its diagonal
elements are very nearly its eigenvalues. At the same
time the matrix E of eigenvectors is developed as the
product of these rotation matrices, E being initialized
as the identity matrix and thereafter multiplied by the
rotation matrix at each step. Because the matrix A is
a 3 x 3 symmetric matrix and the transformations are of
the form $S^t AS$, where S is the orthonormal rotation matrix
and "t" denotes transpose, the symmetry is preserved at
each step so that there are actually only three off-
diagonal elements whose squares need be considered.
The convergence of this version is the fastest of any
Jacobi scheme because it is the largest squared element
which is reduced to zero at each step. The process ends
if either 50 iterations have been performed or if the
sum of the squares of the off-diagonal elements, SSQOD,
is less than 10^{-12} times its initial value.

The intermediate calculations performed by the
routine are to compute the sine and cosine of the ro-
tation angle, because they define the rotation matrix;
rather than do the complete matrix multiplication, only
the nonzero terms in the results of these multiplications
appear in the program. The transformed A-matrix at each
step has six new elements, two reduced to zero, and one
(on the diagonal) unchanged after pre-and-post-multiplying
by the orthogonal rotation matrix and its transpose. Also,
two columns of the E-matrix change at each step and
one remains unchanged. These details all follow the
standard Jacobi analysis as found for example on pages
487-490 of Ref. 4.

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From an argument involving the invariance of the trace of a matrix under a similarity transformation, the fact that the matrix is of dimension three implies that there must be a reduction in SSQOD by a factor not larger than 2/3 at each step (excluding round-off error). Thus if the reduction by a factor of 10^{-12} does not occur by the 50th iteration, the answer is accepted and the analysis in Ref. 1 shows that the reduction (excluding round-off) will have been at worst $(2/3)^{50} \approx 1.568 \times 10^{-9}$. Usually the 10^{-12} reduction factor is reached long before the 50th iteration.

The final section arranges the eigenvalues and eigenvectors so that the eigenvector with the maximum absolute z-component is listed last and the order of the other two vectors is chosen so as to form a right-handed orthonormal system. The eigenvalues are ordered so as conform with that chosen for their corresponding eigenvectors.

The input to this routine is the array A(3,3) containing the three diagonal and three upper triangular elements of the inertia tensor matrix; the three lower triangular elements of the symmetric matrix are inserted at the beginning of EIGENV. The output consists of the three eigenvectors stored in the columns of E(3,3) and the corresponding eigenvalues stored in the diagonal elements A(1,1), A(2,2) and A(3,3).

The variable names in this routine, except for A and E, are not related to any in the other routines. The arrays AA and EE are duplicate copies of the A and E arrays used as temporary storage, as is the variable TEMP. CS, SN, CS2 and SN2 are the cosine and sine of the rotation angle and their squares; IP and IQ are the indices of the pivoted element and NOTPQ is the remaining index which equals neither of these; "SS quantities" are squares or sums of squares; and VLMBDA, VMU and VNU are basically the (Greek letter) variables lambda, mu and nu as in Ref. 4.

3.0 ANALYST'S GUIDE

This chapter contains the mathematical derivations of most of the equations used by MOMENTS-II. Except for 3.5 and 3.6, it is virtually independent of Chapter 2 and is merely a mathematical demonstration of the formulae upon which must of the coding is based. The use of the equations (derived herein) in the computer program is referenced in the Programmer's Guide.

The formulae derived in the first two sections of this chapter are those used in calculations relating to the geometric and physical properties of Prototype Sectors and Standard Wedges; they will be used to compute each of these properties with respect to the (Input) coordinate system as pictured in Fig. 2 and 3. The formulae derived in the next two sections are those which enable one, given the properties of Prototype Sectors and Standard Wedges calculated with respect to those initial coordinate systems, to obtain the properties with respect to translated and/or rotated coordinate systems. The formulae derived in 3.5 and 3.6 enable one to combine the results of sections 3.1 and 3.2, using the rotation and translation equations of sections 3.3 and 3.4 to allow the program to calculate the properties of Angular Wedges and Concave Parts by constructing them from combinations of Prototype Sectors and Standard Wedges.

3.1 PROTOTYPE SECTORS

The purpose of this section is to provide the derivation of the formulae for the various properties of a Prototype Sector (defined in 1.2.1), which is actually an angular sector of a frustum of a right circular cone whose axis coincides with the z-axis (Fig. 1). The notation used in this section for the dimensions of sectors is summarized in 3.1.1, an explanation of the change of variables used for the integrations is found in 3.1.2, and the actual derivations of the expressions for the volume, mass, first moments of inertia, center of gravity and moments and products of inertia for a single Prototype Sector are found in 3.1.3 - 3.1.15.

3.1.1 Notation for Dimension of Sectors

The following table contains the notation used for dimensions of Prototype Sectors in 3.1. A Prototype Sector is illustrated in Fig. 1; the symbols in the figure are not those used here, but rather they are the corresponding FORTRAN symbols, which are indicated below in parentheses. The two sets of symbols are quite similar, although not identical.

SYMBOL	EXPLANATION
D	Minimum z-coordinate of any point of the Prototype Sector (DZ)
H	Length of Prototype Sector, measured parallel to z-axis (H)
t, T	Lower and upper (smaller and larger) angles of Prototype Sector (TH(1), TH(2))
R_1, R_2	Radii at lower-z and upper-z ends of Prototype Sector (R1(1), R2(1))

3.1.2 The Change of Variables for the Integrations

Most of the properties of Prototype Sectors are defined as integrals of the form

$$\iiint f(x,y,z)\,dx\,dy\,dz \tag{1}$$

the region of the integration being the Prototype Sector. However, it is more convenient to change variables before performing the integrations, from the (x,y,z) rectangular coordinates to the (r,R,θ) "conical coordinates" in which r and θ are as in cylindrical coordinates and R is the upper limit on r. ($R=R(z)$ is the radius of the frustum at any z, and it is a linear function of z alone). To obtain the relationship between R and z one can make use of Fig. 10 in which triangles ABC and ADE are similar, so that

$$\frac{R-R_1}{z-D} = \frac{R_2-R_1}{H}$$

which simplifies to

$$z = D + H\,\frac{R-R_1}{R_2-R_1}$$

Because the angle θ is measured from the positive x-axis towards the positive y-axis, the relationship between (x,y) and (r,θ) can be expressed as the usual

$$x = r\cos\theta$$
$$y = r\sin\theta$$

Thus the transformation between the two systems is given by

$$x = r\cos\theta \tag{2a}$$
$$y = r\sin\theta \tag{2b}$$
$$z = D + H\,\frac{(R-R_1)}{R_2-R_1} \tag{2c}$$

In performing the triple integration, the form (1) will be replaced by

$$\iiint f(x(r,R,\theta),y(r,R,\theta),z(r,R,\theta))\,|J|\,dr\,dR\,d\theta \tag{3}$$

In this case the Jacobian J is given by

$$
J = \frac{\partial(x,y,z)}{\partial(r,R,\theta)} = \begin{vmatrix} \cos\theta & 0 & -r\sin\theta \\ \sin\theta & 0 & r\cos\theta \\ 0 & \dfrac{H}{R_2-R_1} & 0 \end{vmatrix} = \frac{-Hr}{R_2-R_1} \tag{4}
$$

The limits on r are 0 to R, and those on θ are t to T, but the limits on R depend on the relative sizes of R_1 and R_2. There are three possible cases to consider, namely $R_1<R_2$, $R_1>R_2$, and $R_1=R_2$. If $R_1<R_2$, then $|R_2-R_1|=R_2-R_1$, so that $|J| = -J$ and the middle integral has limits of R_1 to R_2.

If $R_1>R_2$ then $|R_2-R_1|=-(R_2-R_1)$, so that $|J| = +J$; but also the limits of integration on R are reversed R_2 to R_1 in this case, so that the result is the same as if one had used $-J$ (instead of $|J|$) but had integrated from R_1 to R_2 as in the first case.

Thus the actual form of the integral used for evaluation purposes is

$$
\frac{H}{R_2-R_1} \int_{\theta=t}^{T} \int_{R=R_1}^{R_2} \int_{r=0}^{R} f(x(r,R,\theta),y(r,R,\theta),z(r,R,\theta))\; r\; dr\,dR\,d\theta \tag{5}
$$

The special case of a cylindrical sector $R =R_1=R_2$ can be easily handled by using cylindrical coordinates (r,z,θ) given by

$$
\begin{aligned}
x &= r\cos\theta \\
y &= r\sin\theta \\
z &= z
\end{aligned} \tag{6}
$$

The form of the integrals is then

$$
\int_{\theta=t}^{T} \int_{z=D}^{D+H} \int_{r=0}^{R} f(x(r,z,\theta),y(r,z,\theta),z)\; r\; dr\,dz\,d\theta \tag{7}
$$

where $r= |J|$ and θ is measured as described above. If these integrations are carried out, the resulting formulae are merely specializations to the case $R_1=R_2=R$ of those obtained in 3.2.3 – 3.2.15 using (5); this shows that the expressions derived in the sequel are valid in all cases. (The extension to this case could also be justified using continuity arguments.)

In the remainder of this section the form (5) will be used for evaluating the integrals over a Prototype Sector. The limits of integration on r, R and θ will be omitted: they should be assumed identical with those in (5).

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Except for the calculation of the volume, the (constant) density p will appear in front of each integral, which may be written in the more concise form $\iiint f(x,y,z)dm$, where m is mass and $dm = pdxdydz$.

3.1.3 $V = \iiint dxdydz$ = Volume

The volume V of a Prototype Sector is given by

$$V = \frac{H}{R_2 - R_1} \iiint r \; drdRd\theta$$

which reduces to

$$V = \frac{H(T-t)}{6} [R_2^2 + R_2 R_1 + R_1^2] \tag{8}$$

3.1.4 $m = p\iiint dxdydz$ = Mass

Using (8) one finds the mass of the Prototype Sector to be given by

$$m = pV = \frac{pH(T-t)}{6} [R_2^2 + R_2 R_1 + R_1^2] \tag{9}$$

3.1.5 $Myz = \iiint x dm$ = First moment about yz-plane

The moment Myz of the Prototype Sector is given by

$$Myz = \frac{pH}{R_2 - R_1} \iiint r^2 \cos\theta \; drdRd\theta$$

Performing the integrations, the first moment Myz of the Prototype Sector about the yz-plane is given by

$$Myz = \frac{pH}{12}(\sin T - \sin t)[R_2^3 + R_2^2 R_1 + R_2 R_1^2 + R_1^3]$$

$$\tag{10}$$

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3.1.6 $\underline{Mxz = \iiint y\,dm = \text{First moment about xz-plane}}$

The moment Mxz is given by

$$Mxz = \frac{pH}{R_2 - R_1} \iiint r^2 \sin\theta \, dr\,dR\,d\theta$$

Integrating one finds that the first moment Mxz of the Prototype Sector about the xz-plane is given by

$$Mxz = \frac{pH}{12} (\cos t - \cos T) \, [R_2^3 + R_2^2 R_1 + R_2 R_1^2 + R_1^3] \quad (11)$$

3.1.7 $\underline{Mxy = \iiint z\,dm = \text{First moment about xy-plane}}$

The moment Mxy is given by

$$Mxy = \frac{pH}{R_2 - R_1} \iiint [D + \frac{H(R-R_1)}{R_2-R_1}]\, r \, dr\,dR\,d\theta$$

Breaking this into two integrals, integrating, simplifying and using (9) yields the Moment Mxy of the Prototype Sector, given by

$$Mxy = mD + \frac{pH^2}{24} (T-t) \, [3R_2^2 + 2R_2 R_1 + R_1^2] \quad (12)$$

3.1.8 $\underline{\text{Center of Gravity} = (\bar{x}, \bar{y}, \bar{z})}$

The formulae for the coordinates of the center of gravity of the Prototype Sector are

$$\begin{aligned}
\bar{x} &= Myz/m \\
\bar{y} &= Mxz/m \\
\bar{z} &= Mxy/m,
\end{aligned} \quad (13)$$

where Myz, Mxz, and Mxy are as in 3.1.5-3.1.7. In all cases the coordinates involved will be with respect to the same coordinate system in which the first moments have been expressed.

3.1.9 $\underline{Ix^2 = \iiint x^2\,dm = \text{Second moment about yz-plane}}$

The moment Ix^2 of the Prototype Sector is given by

$$Ix^2 = \frac{pH}{R_2 - R_1} \iiint r^3 \cos^2\theta \, dr\,dR\,d\theta$$

Integrating and using the trigonometric identity

$$\sin(2T) - \sin(2t) = 2\sin(T-t)\cos(T+t),$$

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the second moment Ix^2 of the Prototype Sector about the yz-plane is given by

$$Ix^2 = \frac{pH}{40}[(T-t) + \sin(T-t)\cos(T+t)][R_1^4 + R_2^3R_1 + R_2^2R_1^2 + R_2R_1^3 + R_1^4]$$

(14)

3.1.10 $Iy^2 = \iiint y^2 dm$ = Second moment about the xz-plane

The moment Iy^2 is given by

$$Iy^2 = \frac{pH}{R_2-R_1} \iiint r^3\sin^2\theta \, drdRd\theta$$

Proceeding as in 3.1.9, the second moment Iy^2 of the Prototype Sector about the xz-plane is given by

$$Iy^2 = \frac{pH}{40}[(T-t)-\sin(T-t)\cos(T+t)][R_2^4 + R_2^3R_1 + R_2^2R_1^2 + R_2R_1^3 + R_1^4]$$

(15)

3.1.11 $Iz^2 = \iiint z^2 dm$ = Second moment about the xy-plane

The moment Iz^2 is given by

$$Iz^2 = \frac{pH}{R_2-R_1} \iiint [D + \frac{H(R-R_1)}{R_2-R_1}]^2 r \, drdRd\theta$$

Integration and further simplification yields

$$Iz^2 = \frac{pH^3}{60}(T-t)[6R_2^2 + 3R_2R_1 + R_1^2] + \frac{pH^2D}{12}(T-t)[3R_2^2 + 2R_2R_1 + R_1^2]$$

$$+ \frac{pHD^2}{6}(T-t)[R_2^2 + R_2R_1 + R_1^2]$$

Substituting for m, using (9), adding and subtracting mD^2, and regrouping yields

$$Iz^2 = \frac{pH^3}{60}(T-t)(6R_2^2 + 3R_2R_1 + R_1^2) - mD^2$$

$$+ 2D[mD + \frac{pH^2}{24}(T-t)(3R_2^2 + 2R_2R_1 + R_1^2)]$$

The term in square brackets is the moment Mxy given by (12), so that this reduces to

$$Iz^2 = \frac{pH^3}{60}(T-t)(6R_2^2 + 3R_2R_1 + R_1^2) + 2MxyD - mD^2$$

(16)

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3.1.12 Ixx, Iyy, Izz = Moments of inertia about the axes

$$Ixx = \iiint (y^2 + z^2)dm = \text{moment of inertia about x-axis}$$

$$Iyy = \iiint (x^2 + z^2)dm = \text{moment of inertia about y-axis}$$

$$Izz = \iiint (x^2 + y^2)dm = \text{moment of inertia about z-axis}$$

These quantities are evaluated for a Prototype Sector using the results (14), (15), and (16) as follows:

$$Ixx = Iy^2 + Iz^2$$

$$Iyy = Ix^2 + Iz^2 \tag{17}$$

$$Izz = Ix^2 + Iy^2$$

3.1.13 Ixz = $-\iiint xzdm$ = xz-product of inertia

The product of inertia Ixz of the Prototype Sector is given by

$$Ixz = \frac{-pH}{R_2 - R_1} \iiint r^2\cos\theta \left[D + \frac{H(R - R_1)}{R_2 - R_1} \right] drdRd\theta$$

Breaking this into two parts, integrating and simplifying yields the product of inertia Ixz of the Prototype Sector, given by

$$Ixz = \frac{-pH}{60}(\sin T - \sin t)[5D(R_2^3 + R_2^2 R_1 + R_2 R_1^2 + R_1^3)$$

$$+ H(4R_2^3 + 3R_2^2 R_1 + 2R_2 R_1^2 + R_1^3)] \tag{18}$$

3.1.14 Iyz = $-\iiint yzdm$ = yz-product of inertia

The product of inertia Iyz is given by

$$Iyz = \frac{-pH}{R_2 - R_1} \iiint r^2\sin\theta \left[D + \frac{H(R - R_1)}{R_2 - R_1} \right] drdRd\theta$$

Proceeding as in 3.2.13, one obtains the product of inertia Iyz of the Prototype Sector, given by

$$Iyz = \frac{-pH}{60}(\cos t - \cos T)[5D(R_2^3 + R_2^2 R_1 + R_2 R_1^2 + R_1^3)$$

$$+ H(4R_2^3 + 3R_2^2 R_1 + 2R_2 R_1^2 + R_1^3)] \tag{19}$$

3.1.15 $Ixy = -\iiint xy\,dm$ = xy-product of inertia

The product of inertia Ixy given by

$$Ixy = \frac{-pH}{R_2 - R_1} \iiint r^3 \sin\theta\cos\theta \, dr\,dR\,d\theta$$

Performing the integrations, and some trigonometric simplifications, one finds the product of inertia Ixy of the Prototype Sector to be given by

$$Ixy = \frac{-pH}{40} (\cos^2 t - \cos^2 T) [R_2^4 + R_2^3 R_1 + R_2^2 R_1^2 + R_2 R_1^3 + R_1^4]$$

(20)

3.2 STANDARD WEDGES

The purpose of this section is to provide the derivations of the formulae for the various properties of the Standard Wedge, which is actually either a complete right angle wedge or one which has been sliced (truncated) by a plane parallel to the xy-plane. The Wedge is assumed to be positioned as indicated in Figure 3. The notation used here for the dimensions of the Standard Wedge is summarized in 3.2.1, an explanation of the change of variables used for the integrations is found in 3.2.2, and the actual derivations of the expressions for the volume, mass, first moments of inertia, center of gravity, and moments and products of inertia for a Standard Wedge are found in 3.2.3 - 3.2.15.

3.2.1 Notations for Dimensions of Standard Wedges

The following table contains the notation used for dimensions of Standard Wedges in 3.2. A Standard Wedge is illustrated in Figure 3; the (FORTRAN) symbols used in the figure are shown below in parentheses after the explanation of the corresponding symbol used in this section.

SYMBOL	EXPLANATION
D	Minimum z-coordinate of any point of the Standard Wedge (DZ)
H	Length of Standard Wedge, measured parallel to z-axis (H)
W	Width of Standard Wedge, measured parallel to y-axis (W)
R_1, R_2	Heights of Standard Wedge at its lower-z and upper-z faces, measured parallel to the x-axis (RR1, RR2)

3.2.2 Change of Variables for Integrations

As was the case for Prototype Sectors, it is convenient to change variables when integrating over a Standard Wedge. The coordinates used here will be (x, y, R), where R is identically the same R discussed in 3.1.2 and is related to z according to (2c). The reason that this same variable is convenient for Wedges also is that the (side) cross-sectional view (xz-plane) of Standard Wedges is identical to that in the plane formed by the z-axis and any coplanar straight line on the surface of the Prototype Sector. This means that at any fixed z-value, the height of the Standard Wedge is the same as the radius of the Prototype Sector (assuming, of course, that both objects have the same D, H, R_1 and R_2). Thus both Figure 10 and the previous derivation for R as a function of z apply here also. The x and y coordinates, however, need not be replaced because the yz-plane cross section of a Wedge is a rectangle. Therefore, the transformation of coordinates used here is given by

$$x = x$$
$$y = y$$
$$z = D + H \frac{(R-R_1)}{(R_2-R_1)} \tag{21}$$

In performing the triple integrations, the form used will be

$$\iiint f(x,y,z(R)) |J| \, dxdydR \tag{22}$$

In this case the Jacobian J is given by

$$J = \frac{\partial(x,y,z)}{\partial(x,y,R)} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{H}{R_2-R_1} \end{vmatrix} = \frac{H}{R_2-R_1} \tag{23}$$

Following the same line of reasoning as in 3.1.2, for the first two cases, one can obtain the actual form of the integrals used in the sequel, which is

$$\frac{H}{R_2-R_1} \int_{R=R_1}^{R_2} \int_{y=-W/2}^{+W/2} \int_{x=0}^{R} f(x,y,z(R)) \, dxdydR \tag{24}$$

Here, the special case of a rectangular parallelepiped $R=R_1=R_2$ can be handled easily by using ordinary rectangular coordinates in which case the form of the integrals becomes

$$\int_{z=D}^{D+H} \int_{y=-W/2}^{+W/2} \int_{x=0}^{R} f(x,y,z) \, dxdydz \tag{25}$$

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As in 3.1, if these integrations are carried out (for this special case) the resulting formulas are merely specializations to this case of those formulas obtained in 3.2.3 - 3.2.15 using (24); this shows that the expressions derived in what follows are valid in all cases. Here also, the limits of integration will be omitted henceforth and those in (24) should be assumed. Finally, as in 3.1, the shortened notation $\iiint f(x,y,z)dm$ will be used in the sub-section headings.

3.2.3 $V = \iiint dxdydz = $ Volume

Since the xz-cross section of the Wedge is a trapezoid with bases R_1 and R_2 and height H, the cross sectional area is given by

$$A = \frac{H}{2} (R_1 + R_2)$$

Since this cross section is constant and the width is W, the volume is given by

$$V = \frac{WH}{2} (R_1 + R_2) \tag{26}$$

3.2.4 $m = \iiint p\,dxdydz = $ Mass

Since the density p is constant throughout the Wedge, its mass is given by $m = pV$ or

$$m = \frac{pWH}{2} (R_1 + R_2) \tag{27}$$

3.2.5 $Myz = \iiint x\,dm = $ First moment of inertia about the yz-plane

The moment Myz is given by

$$Myz = \frac{pH}{R_2 - R_1} \iiint x \, dxdydR$$

Performing the integrations one obtains the formula for the first moment Myz of the Standard Wedge

$$Myz = \frac{pHW}{6}(R_2^2 + R_2 R_1 + R_1^2) \tag{28}$$

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3.2.6 Mxy = $\iiint z\,dm$ = First moment of inertia about the xy-plane

The first moment Mxy is given by

$$Mxy = \frac{pH}{R_2-R_1} \iiint \left[D + H\ \frac{(R-R_1)}{R_2-R_1}\right]\ dx\,dy\,dR$$

Breaking this into two integrals, integrating and simplifying yields the first moment of inertia Mxy of the Standard Wedge, given by

$$Mxy = \frac{pHW}{6}\ \left[(2R_2 + R_1)H + 3D(R_1+ R_2)\right] \tag{29}$$

3.2.7 Mxz = $\iiint y\,dm$ = First moment of inertia about the xz-plane

Because of the symmetry of the Standard Wedge about the xz-plane, it is true that

$$Mxz = 0 \tag{30}$$

This could also be seen by noting that in this case the integral reduces to (24) with integrand $\underline{y}$, so that the middle integral will yield a zero result.

3.2.8 Center of Gravity = $(\bar{x},\bar{y},\bar{z})$

The formulae for the coordinates of the center of gravity of the Standard Wedge are

$$\bar{x} = Myz/m$$

$$\bar{y} = Mxz/m = 0 \tag{31}$$

$$\bar{z} = Mxy/m$$

where Myz, Mxz and Mxy are given by (28), (29) and (30). These coordinates will be with respect to the same coordinate system in which the first moments have been expressed.

3.2.9 $Ix^2 = \iiint x^2\,dm$ = Second moment of inertia about the yz-plane

This second moment can be expressed as

$$Ix^2 = \frac{pH}{R_2-R_1} \iiint x^2\ dx\,dy\,dR$$

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From this one obtains the expression for the second moment Ix^2 of the Standard Wedge about the yz-plane.

$$Ix^2 = \frac{pHW}{12}(R_2^3 + R_2^2 R_1 + R_2 R_1^2 + R_1^3) \qquad (32)$$

3.2.10 $Iy^2 = \iiint y^2 dm$ = Second moment of inertia about the xz-plane

This second moment can be expressed as

$$Iy^2 = \frac{pH}{R_2 - R_1} \iiint y^2 \, dxdydR$$

which reduces to

$$Iy^2 = \frac{pHW^3}{24}(R_1 + R_2) \qquad (33)$$

3.2.11 $Iz^2 = \iiint z^2 dm$ = Second moment of inertia about the xy-plane

This second moment can be expressed as

$$Iz^2 = \frac{pH}{R_2 - R_1} \iiint [D + \frac{H(R - R_1)}{R_2 - R_1}]^2 \, dxdydR$$

Breaking this into three parts, integrating and simplifying yields the formula for the second moment Iz^2 of the Standard Wedge about the xy-plane,

$$Iz^2 = \frac{pWH}{12}[H^2(3R_2 + R_1) + 4DH(2R_2 + R_1) + 6D^2(R_1 + R_2)] \qquad (34)$$

3.2.12 Ixx, Iyy, Izz = Moments of inertia about the axes

$Ixx = \iiint (y^2 + z^2) dm$ = moment of inertia about the x-axis.
$Iyy = \iiint (x^2 + z^2) dm$ = moment of inertia about the y-axis.
$Izz = \iiint (x^2 + y^2) dm$ = moment of inertia about the z-axis.

The quantities are evaluated for a Standard Wedge using the results (32), (33) and (34) as follows:

$$\begin{aligned} Ixx &= Iy^2 + Iz^2 \\ Iyy &= Ix^2 + Iz^2 \\ Izz &= Ix^2 + Iy^2 \end{aligned} \qquad (35)$$

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3.2.13 $Ixz = -\iiint xz\,dm = $ xz-product of inertia

This product of inertia is given by

$$Ixz = \frac{-\rho H}{R_2 - R_1} \iiint x[D + \frac{H(R-R_1)}{R_2 - R_1}]dxdydR$$

Integrating directly, or simply using the similarity to the integral in 3.1.7 using (12) and (9), one obtains the Ixz product of inertia of the Standard Wedge

$$Ixz = \frac{-\rho HW}{24}[4D(R_2^2 + R_2R_1 + R_1^2) + H(3R_2^2 + 2R_2R_1 + R_1^2)] \quad (36)$$

3.2.14 $Iyz = -\iiint yz\,dm = $ yz-product of inertia

Because of the symmetry of the Standard Wedge about the xz-plane, it is true that

$$Iyz = 0 \quad (37)$$

This could also be seen by noting the presence of y to an even power in the integrand, which causes the second integrating to yield a zero result.

3.2.15 $Ixy = -\iiint xy\,dm = $ xy-product of inertia

Because of the symmetry of the Standard Wedge about the xz-plane

$$Ixy = 0 \quad (38)$$

This result could also be obtained as in 3.2.14.

3.3 TRANSLATIONS

This section contains the derivations of the equations for transforming the moments and products of inertia as well as the center of gravity (calculated using the formulas and methods of 3.1 - 3.2 with respect to an initial coordinate system) under a translation, to a new coordinate system which is parallel to the first but has a different origin. Coordinates and quantities referred to the initial system only will be denoted by the subscript or superscript "o" (and coordinates and quantities in the translated system will not). This transformation can be completely described by specifying the three scalers (x_p, y_p, z_p) which are the new system coordinates of the origin of the old system; the corresponding change of variables is then given by

$$x = x_o + x_p$$

$$y = y_o + y_p \tag{39}$$

$$z = z_o + z_p$$

In changing coordinates, it should be noted that the Jacobian J of this transformation is given by the determinant

$$J = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \tag{40}$$

as expected, since a translation alone does not change volumes at all.

The next three subsections contain derivations of the expressions for the coordinates of the center of gravity, and for the moments and products of inertia in a translated system, in terms of the center of gravity, moments, and products of inertia in the initial system.

3.3.1 Center of Gravity Translation

If the coordinates of the center of gravity of an object are denoted by $(\bar{x}, \bar{y}, \bar{z})$ and $(\bar{x}_o, \bar{y}_o, \bar{z}_o)$ in the new and old systems respectively, then using (39) one obtains that the new coordinates of the center of gravity are given in

terms of the old by

$$\bar{x} = \bar{x}_o + x_p$$
$$\bar{y} = \bar{y}_o + y_p \qquad\qquad (41)$$
$$\bar{z} = \bar{z}_o + z_p$$

3.3.2 Moment translation

The transformation corresponding to a translation of coordinates from an initial system (denoted by subscript or superscript "o") to a new system, expressing the new moments in terms of initial system quantities, is given by

$$Ixx = \overset{o}{I}xx + m[2y_p\bar{y}_o + 2z_p\bar{z}_o + (y_p^2 + z_p^2)]$$
$$Iyy = \overset{o}{I}yy + m[2x_p\bar{x}_o + 2z_p\bar{z}_o + (x_p^2 + z_p^2)] \qquad (42)$$
$$Izz = \overset{o}{I}zz + m[2x_p\bar{x}_o + 2y_p\bar{y}_o + (x_p^2 + y_p^2)]$$

where the notation is the same as in 1.1 and 3.1.

The first of these formulas will be demonstrated, derivations for the others being entirely analogous. In all cases the region of integration is the same and the limits in the different coordinate systems are chosen accordingly.

By definition, $\overset{o}{I}xx \equiv \iiint (y_o^2 + z_o^2)\,dm_o$.

Writing down the definition of Ixx and performing the change of variables (39) using (40) yields

$$Ixx \equiv \iiint (y^2+z^2)\,dm = \iiint [(y_o+y_p)^2 + (z_o+z_p)^2]\,|J|\,dm_o$$

$$= \iiint (y_o^2+z_o^2)\,dm_o + 2y_p\iiint y_o\,dm_o + 2z_p\iiint z_o\,dm_o + (y_p^2+z_p^2)\iiint dm_o$$

$$= \overset{o}{I}xx + 2y_p(m\bar{y}_o) + 2z_p(m\bar{z}_o) + m(y_p^2+z_p^2)$$

which reduces to the first of (42).

As before $dm = pdxdydz$ and similarly here the notation $dm_o = pdx_ody_odz_o$ is used to indicate the coordinate system. Obviously $dm = |J|dm_o = dm_o$ and the quantity m_o representing the mass in the initial coordinate system is never used since $m=m_o$.

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To write this in terms of new coordinates of the center of gravity, one substitutes for $\overline{x}_o, \overline{y}_o$, and $\overline{z}_o$ using (41) and simplifies, to obtain

$$Ixx = I\overset{\circ}{x}x + m[y_p(2\overline{y}-y_p) + z_p(2\overline{z}-z_p)] \tag{43}$$

The forms represented by (43) and (45) express the new quantities in terms of old quantities, except that they are in terms of the new coordinates of the center of gravity. It is these forms of the equations which are used by Subroutine TRANSL, which applies equation (41) before applying either (43) or (45).

3.3.3 Product Translations

The transformation of the products of inertia corresponding to a translation of coordinates from the initial ("o" system) to the new system can be expressed as

$$Ixy = I\overset{\circ}{x}y - m(x_p\overline{y}_o + y_p\overline{x}_o + x_py_p)$$

$$Ixz = I\overset{\circ}{x}z - m(x_p\overline{z}_o + z_p\overline{x}_o + x_pz_p) \tag{44}$$

$$Iyz = I\overset{\circ}{y}z - m(y_p\overline{z}_o + z_p\overline{y}_o + y_pz_p)$$

where the notation is as before.

Again, only the first of these will be demonstrated, the others being entirely analogous, and the region of integration is fixed.

By definition $I\overset{\circ}{x}y = -\iiint x_oy_odm_o$.

Writing down the definition of Ixy and performing the change of variables (39) using (40), yields

$$Ixy \equiv -\iiint xydm = -\iiint(x_o+x_p)(y_o+y_p)\ |J|dm_o$$

$$= -\iiint x_oy_odm_o - x_p\iiint y_odm_o - y_p\iiint x_odm_o - x_py_p\iiint dm_o$$

$$= I\overset{\circ}{x}y - x_p(m\overline{y}_o) - y_p(m\overline{x}_o) - mx_py_p$$

which reduces to the first of (44).

To write this in terms of new coordinates of the center of gravity, one substitutes for $\overline{x}_o$, $\overline{y}_o$ and $\overline{z}_o$ using (41) and simplifies to obtain

$$Ixy = I\overset{\circ}{x}y - m(x_p\overline{y}+y_p\overline{x}-x_py_p) \tag{45}$$

This is the form of the equation used by subroutine TRANSL.

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3.4 ROTATIONS

This section contains the derivations of the equations for transforming the moments and products of inertia as well as the center of gravity (calculated using the formulae in 3.1 - 3.2 with respect to an initial coordinate system) under a rotation to a new coordinate system. This transformation can be completely described by specifying the set of nine direction cosines (a_i, b_i, c_i) for $i=1$, 2, 3 of the (initial) old system axes with respect to the new. Thus, if $\vec{i}_0, \vec{j}_0, \vec{k}_0$ are unit vectors along the <u>initial</u> system x_0, y_0 and z_0 axes, and $\vec{i}, \vec{j}, \vec{k}$ are unit vectors along the new system x, y, and z axes, the rotation can be expressed as

$$\vec{i}_0 = a_1 \vec{i} + b_1 \vec{j} + c_1 \vec{k}$$

$$\vec{j}_0 = a_2 \vec{i} + b_2 \vec{j} + c_2 \vec{k} \qquad (46)$$

$$\vec{k}_0 = a_3 \vec{i} + b_3 \vec{j} + c_3 \vec{k}$$

Equivalently, the transformation used to express this rotation in terms of the old and new <u>coordinates</u> is

$$x = a_1 x_0 + a_2 y_0 + a_3 z_0$$

$$y = b_1 x_0 + b_2 y_0 + b_3 z_0 \qquad (47)$$

$$z = c_1 x_0 + c_2 y_0 + c_3 z_0$$

This can be seen by writing any point P (x_0, y_0, z_0) as a vector in the old system as $\vec{P} = x_0 \vec{i}_0 + y_0 \vec{j}_0 + z_0 \vec{k}_0$, substituting for $\vec{i}_0, \vec{j}_0$ and $\vec{k}_0$ in terms of $\vec{i}, \vec{j}$ and $\vec{k}$ using (46), and equating the coefficients of $\vec{i}, \vec{j}$, and $\vec{k}$ between this and the representation for the same point $P = (x,y,z)$ in new coordinates as $\vec{P} = x\vec{i} + y\vec{j} + z\vec{k}$. Equation (47) can be expressed more concisely as follows. If $\vec{V}_0 = \langle x_0, y_0, z_0 \rangle$ are the initial systems components of a vector, then the new system components of the same vector V are given by

$$\vec{V} = R\vec{V}_0, \qquad (48a)$$

where

$$R = \begin{matrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{matrix} \qquad (48b)$$

In performing the change of coordinates (47), the Jacobian J of this transformation is given by the determinant

$$J = Det(R) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 1 \qquad (49)$$

as expected, since a rotation alone does not change volumes in any way.

The next sub-section provides some preliminary facts about direction cosines; these are needed for the derivations of the expressions for the coordinates of the center of gravity and the moments and products in the rotated system in terms of old system quantities which are presented in 3.4.3 - 3.4.4.

3.4.1 Direction Cosines

The quantities (a_i, b_i, c_i) for i=1,2,3 referred to in 3.4 are called direction cosines. If the angles between the (old) $\vec{i}_o$ vector and the new $\vec{i}, \vec{j}, \vec{k}$ vectors are A_1, B_1, C_1, those between $\vec{j}_o$ and these axes are A_2, B_2, C_2, and those between $\vec{k}_o$ and these axes are A_3, B_3, C_3, then the following relationship holds between the angles and the a_i, b_i and c_i's:

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} = \begin{pmatrix} \cos A_1 & \cos A_2 & \cos A_3 \\ \cos B_1 & \cos B_2 & \cos B_3 \\ \cos C_1 & \cos C_2 & \cos C_3 \end{pmatrix}$$

In the sequel, the shorter (a_i, b_i, c_i) notation will be employed in place of the more cumbersome "cosines notation".

By inverting (46) one can also express the new unit vectors in terms of old as

$$\vec{i} = a_1 \vec{i}_o + a_2 \vec{j}_o + a_3 \vec{k}_o$$

$$\vec{j} = b_1 \vec{i}_c + b_2 \vec{j}_o + b_3 \vec{k}_o \qquad (50)$$

$$\vec{k} = c_1 \vec{i}_o + c_2 \vec{j}_o + c_3 \vec{k}_o$$

One can now use (46) and (50) to derive the sets of relations among the direction cosines to be used in 3.4.3 - 3.4.4.

By noting the vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors and using (46) one obtains

$$a_1^2 + b_1^2 + c_1^2 = 1$$

$$a_2^2 + b_2^2 + c_2^2 = 1 \qquad (51)$$

$$a_3^2 + b_3^2 + c_3^2 = 1$$

Taking dot products of the vectors with each other (noting that the initial system is an orthogonal coordinate system), one obtains

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$a_1 a_3 + b_1 b_3 + c_1 c_3 = 0 \qquad (52)$$

$$a_2 a_3 + b_2 b_3 + c_2 c_3 = 0$$

Similarly, the same two observations about the new system using (50) yield

$$a_1^2 + a_2^2 + a_3^2 = 1$$

$$b_1^2 + b_2^2 + b_3^2 = 1 \qquad (53)$$

$$c_1^2 + c_2^2 + c_3^2 = 1$$

and

$$a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$$

$$a_1 c_1 + a_2 c_2 + a_3 c_3 = 0 \qquad (54)$$

$$b_1 c_1 + b_2 c_2 + b_3 c_3 = 0$$

3.4.2 Center of Gravity Rotation

If the coordinates of the center of gravity of an object are denoted by $(\bar{x}, \bar{y}, \bar{z})$ and $(\bar{x}_o, \bar{y}_o, \bar{z}_o)$ in the new and old systems, respectively, then (47) yields the new coordinates of the center of gravity given by

$$\bar{x} = a_1 \bar{x}_o + a_2 \bar{y}_o + a_3 \bar{z}_o$$

$$\bar{y} = b_1 \bar{x}_o + b_2 \bar{y}_o + b_3 \bar{z}_o \qquad (55)$$

$$\bar{z} = c_1 \bar{x}_o + c_2 \bar{y}_o + c_3 \bar{z}_o$$

3.4.3 Moment Rotations

The transformation corresponding to the rotation of coordinates (47) from an initial (old) system (denoted by subscript or superscript "o") to a new system, expressing the new moments of inertia in terms of initial system quantities is given by

$$Ixx = a_1^2 I\overset{o}{x}x + a_2^2 I\overset{o}{y}y + a_3^2 I\overset{o}{z}z + 2a_1 a_2 I\overset{o}{x}y + 2a_1 a_3 I\overset{o}{x}z +$$

$$2a_2 a_3 I\overset{o}{y}z$$

$$Iyy = b_1^2 I\overset{o}{x}x + b_2^2 I\overset{o}{y}y + b_3^2 I\overset{o}{z}z + 2b_1 b_2 I\overset{o}{x}y + 2b_1 b_3 I\overset{o}{x}z +$$

$$2b_2 b_3 I\overset{o}{y}z \qquad (56)$$

$$Izz = c_1^2 I\overset{o}{x}x + c_2^2 I\overset{o}{y}y + c_3^2 I\overset{o}{z}z + 2c_1 c_2 I\overset{o}{x}y + 2c_1 c_3 I\overset{o}{x}z +$$

$$2c_2 c_3 I\overset{o}{y}z$$

Only the first of these formulae will be demonstrated; derivations for the others, being entirely analogous, will not be presented here. (The entire resultant rotation effect is summarized in 3.4.5).

The basic result comes from the definition of Ixx and the rule for changing variables in multiple integrals. Thus

$$Ixx \equiv \iiint (y^2 + z^2)\,dm \equiv \iiint \{[y(x_o, y_o, z_o)]^2 + [z(x_o, y_o, z_o)]^2\}|J|\,dm_o$$

Substituting (47) and (49) into this equation yields

$$I_{xx} = \iiint [b_1^2 x_o^2 + b_2^2 y_o^2 + b_3^2 z_o^2 + c_1^2 x_o^2 + c_2^2 y_o^2 + c_3^2 z_o^2$$

$$+ 2b_1 b_2 x_o y_o + 2b_1 b_3 x_o z_o + 2b_2 b_3 y_o z_o$$

$$+ 2c_1 c_2 x_o y_o + 2c_1 c_3 x_o z_o + 2c_2 c_3 y_o z_o] dm_o$$

Using the definitions of $I\overset{o}{x}^2$, $I\overset{o}{y}^2$, $I\overset{o}{z}^2$, $I\overset{o}{x}y$, $I\overset{o}{x}z$ and $I\overset{o}{y}z$ (see 1.1.1) to remove the integral signs yields

$$I_{xx} = (b_1^2 + c_1^2) I\overset{o}{x}^2 + (b_2^2 + c_2^2) I\overset{o}{y}^2 + (b_3^2 + c_3^2) I\overset{o}{z}^2$$

$$-2(b_1 b_2 + c_1 c_2) I\overset{o}{x}y \;\; -2(b_1 b_3 + c_1 c_3) I\overset{o}{x}z \;\; -2(b_2 b_3 + c_2 c_3) I\overset{o}{y}z$$

Using (51), (52) and the first of (53) to substitute for the quantities in parentheses yields

$$I_{xx} = (a_2^2 + a_3^2) I\overset{o}{x}^2 + (a_1^2 + a_3^2) I\overset{o}{y}^2 + (a_1^2 + a_2^2) I\overset{o}{z}^2$$

$$+ 2a_1 a_2 I\overset{o}{x}y + 2a_1 a_3 I\overset{o}{x}z + 2a_2 a_3 I\overset{o}{y}z$$

Regrouping and using the definitions of $I\overset{o}{x}x$, $I\overset{o}{y}y$, and $I\overset{o}{z}z$ yields the required first equation of (56) which expresses the moment of inertia I_{xx} in the new system as a function of the moments and products of inertia in the old coordinate system and the direction cosines of the new $\vec{1}$ axis.

3.4.4 Product Rotations

The transformation of the products of inertia corresponding to a rotation of coordinates from the initial ("o" system) to the new system can be expressed as

$$I_{xy} = a_1 b_1 I\overset{o}{x}x + a_2 b_2 I\overset{o}{y}y + a_3 b_3 I\overset{o}{z}z$$

$$+ (a_1 b_2 + b_1 a_2) I\overset{o}{x}y + (a_1 b_3 + b_1 a_3) I\overset{o}{x}z + (a_2 b_3 + b_2 a_3) I\overset{o}{y}z$$

$$I_{xz} = a_1 c_1 I\overset{o}{x}x + a_2 c_2 I\overset{o}{y}y + a_3 c_3 I\overset{o}{z}z \qquad\qquad (57)$$

$$+ (a_1 c_2 + c_1 a_2) I\overset{o}{x}y + (a_1 c_3 + c_1 a_3) I\overset{o}{x}z + (a_2 c_3 + c_2 a_3) I\overset{o}{y}z$$

$$I_{yz} = b_1 c_1 I\overset{o}{x}x + b_2 c_2 I\overset{o}{y}y + b_3 c_3 I\overset{o}{z}z$$

$$+ (b_1 c_2 + c_1 b_2) I\overset{o}{x}y + (b_1 c_3 + c_1 b_3) I\overset{o}{x}z + (b_2 c_3 + c_2 b_3) I\overset{o}{y}z,$$

where the notation is as before.

Again, only the first of these will be demonstrated, the others being entirely analogous. The basic result comes from the definition of Ixy. Thus,

$$Ixy \equiv -\iiint xy\,dm = -\iiint \{ x(x_O, y_O, z_O) y(x_O, y_O, z_O) \} |J|\,dm_O$$

Substituting into this using (47) and (49) yields

$$Ixy = -\iiint [a_1 b_1 x_O^2 + a_2 b_2 y_O^2 + a_3 b_3 z_O^2 + a_1 b_2 x_O y_O + a_1 b_3 x_O z_O$$
$$+ a_2 b_3 y_O z_O + b_1 a_2 x_O y_O + b_1 a_3 x_O z_O + b_2 a_3 y_O z_O]\,dm_O$$

Using the definitions of $I\overset{o}{x}^2$, $I\overset{o}{y}^2$ etc., this becomes

$$Ixy = -a_1 b_1 I\overset{o}{x}^2 - a_2 b_2 I\overset{o}{y}^2 - a_3 b_3 I\overset{o}{z}^2 + (a_1 b_2 + b_1 a_2) I\overset{o}{x}y$$
$$+ (a_1 b_3 + b_1 a_3) I\overset{o}{x}z + (a_2 b_3 + b_2 a_3) I\overset{o}{y}z$$

Applying the first of (54) to the coefficients of the first three terms yields

$$Ixy = (a_2 b_2 + a_3 b_3) I\overset{o}{x}^2 + (a_1 b_1 + a_3 b_3) I\overset{o}{y}^2 + (a_1 b_1 + a_2 b_2) I\overset{o}{z}^2$$
$$+ (a_1 b_2 + b_1 a_2) I\overset{o}{x}y + (a_1 b_3 + b_1 a_3) I\overset{o}{x}z + (a_2 b_3 + b_2 a_3) I\overset{o}{y}z$$

Regrouping and using the definitions of $I\overset{o}{x}x$, $I\overset{o}{y}y$ and $I\overset{o}{z}z$ results in the required first equations of (57), which expresses the product of inertia Ixy in the new system as a function of the moments and products of inertia in the old coordinate system and of the direction cosines of the $\vec{i}$ and $\vec{j}$ axes.

3.4.5 Rotations Revisited

Sub-Sections 3.4.3 and 3.4.4 show how one can derive the expressions for the moments and products of inertia with respect to a new (rotated) coordinate system, in terms of those with respect to an initial coordinate system. These derivations were presented so as to require a minimal amount of knowledge to understand; yet they do not resort to overlooking basic facts about general rotations (and to taking the naive approach of decomposing a compound rotation into a sequence of simple rotations about single axes). The following briefly outlines how one can obtain the same results simply in another fashion, using a more elegant argument requiring a slight knowledge of tensors.

If one uses matrix notation, one can obtain a concise and revealing method of expressing relations (56) and (57)

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in a single formula. The inertia tensor $\bar{I}$ is defined as the tensor of rank two

$$\bar{I} = \begin{array}{ccc} Ixx & Ixy & Ixz \\ Ixy & Iyy & Iyz \\ Ixz & Iyz & Izz \end{array} \qquad (58)$$

and the inertia tensor in the original coordinate system $\bar{I}o$ is defined similarly, with "o" superscripted quantities Ixx, Ixy etc. appearing as elements. A combination of the six equations (56) and (57) can be written as

$$\bar{I} = R\bar{I}_O R^t \qquad (59)$$

where $\bar{I}$ and $\bar{I}_O$ are as above, R is given by (48) and "t" denotes transpose. If one is willing to accept the fact that the entity defined in (58) is a (contravariant) tensor of rank two, then by definition it transforms under any one-to-one change of coordinates as

$$(\bar{I})^{mn} = \frac{\partial x_m}{\partial x_i^o} \frac{\partial x_n}{\partial x_j^o} (\bar{I}_O)^{ij} \qquad (60)$$

where the integers outside the parentheses are component indices (two indices in three-dimensional space yield a total of nine components for the second rank tensor), "o" indicates initial system coordinates, and all indices appearing twice in any one term (namely i and j) must be summed from 1 to 3. In the present case, the matrix of values for $\frac{\partial x_m}{\partial x_i^o}$ for i,m=1,...,3 is merely R, given by (48b), and the matrix of values $(\bar{I})^{ij}$ for i,j=1,...,3 is given by (58). Substituting these values into (60) one will obtain the nine equations which are summarized in the concise form (59), six of which are distinct and are given by (56) and (57), as required.

The familiarity of the form of equation (59) is not a coincidence, as can be seen by re-phrasing the above tensor formulation as follows: whereas (50) is the standard method for linearly transforming (contravariant) tensors of rank one (i.e., vectors), the inertia tensor is a (contravariant) tensor of rank two and in the case where this transformation is a rotation only it transforms the same way as a matrix. That is, the rule for transforming $\bar{I}_O$ to $\bar{I}$ under a pure rotation is the special case of the familiar $\bar{I} = R\bar{I}_O R^{-1}$, where the matrix R is orthonormal so that $R^{-1} = R^t$ and (59) is obtained.

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3.5 ANGULAR WEDGES

The purpose of this section is to provide a detailed explanation of the method of calculating the properties of an "Angular Wedge", which is actually a triangular solid such that its (triangular) ends are parallel to the xy-plane and the line of intersection of two of its sides lies on the z-axis. The notation used in this section for the dimensions of the Angular Wedge is summarized in 3.5.1, the method of representing the Angular Wedge as a combination of two Standard Wedges is summarized in 3.5.2, and the derivations of the detailed descriptions of these Standard Wedges for the two possible cases are provided in 3.5.3 and 3.5.4 thus completing the description of the method (these results form the basis of subroutine WEDGE1, described in 2.4).

3.5.1 Notation for Dimensions of Angular Wedges

This table contains the notation used for dimensions of Angular Wedges in section 3.5. An Angular Wedge is illustrated in Figure 4; the (FORTRAN) symbols used in the figure are shown below in parentheses.

SYMBOL	EXPLANATION
D	Minimum z-coordinate of any point on Angular Wedge, measured with respect to its own Input Axes (DZ)
H	Length of Angular Wedge, measured parallel to its Input z-axis (H)
t,T	Lower and upper angles of Angular Wedge (TH(1), TH(2))
θ	T-t (TH(2)-TH(1))
R_1, R_2	Radii at lower and upper angles of Angular Wedge (RR1,RR2)

3.5.2   Decomposing the Angular Wedge

The method of calculating the properties of an Angular Wedge amounts to geometrically modelling it, or treating it as a combination of two Standard Wedges whose properties are calculated (with respect to their own Input Systems) using the method outlined in 3.2.   The method of representing an Angular Wedge as a sum or difference of two Standard Wedges is based on the following observation: By constructing the altitude plane (EFGH in Fig. 11) from the edge of the Angular Wedge which lies on the z-axis to face ABCD of the Angular Wedge, one can create two "component right-angle wedges" such that:

a)   If the altitude falls inside the Angular Wedge, then the two newly formed right-angle wedges will both be interior to the Angular Wedge, so that it can be represented as their <u>sum</u>.

b)   If the altitude falls outside the Angular Wedge requiring the extension of face ABCD (as in Fig. 11), one of the two newly formed right-angle wedges will lie totally exterior to the Angular Wedge and the other will contain both it and the Angular Wedge, in which case the Angular Wedge can be represented as the <u>difference</u> of the larger and the smaller right-angle Wedges.

The application of this observation is that in either case, each of these two "component right-angle wedges" can be considered as a Standard Wedge (the special cases $R_1=0$ or $R_2=0$ only, because they have five faces instead of the more general six) and can be described to the program (as explained in 2.4), which can then calculate their properties with respect to their Input Systems using the method described in 3.2.   (The fact that the Input System of these Standard Wedges will in general be different from the Input System of the Angular Wedges under consideration implies that these properties will have to be rotated and/ or translated by using the methods described in 3.3 - 3.4.)

This is the desired modelling of an Angular Wedge into Standard Wedges, which, as is apparent from the above re-marks, falls naturally into two cases according as the Angular Wedge is to be expressed as a <u>sum</u> or as a <u>difference</u> of the two Standard Wedges.   The remainder of this sub-section contains derivations of expressions for certain quantities which are independent of which case is being con-sidered, and it lays the groundwork for the detailed dis-cussions of the two cases which can be found in the next two subsections.

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Since the xy-cross section of the Angular Wedge is constant, only that one cross section will be considered in what follows; that is, without loss of generality, expressions will be derived for the relevant quantities describing the two "Sub-Wedges" using a two-dimensional approach confined (almost) strictly to the xy-plane. Thus, triangle AOB will be considered and con tructing its altitude PO from side AB to the vertex at O, the two cases can be characterized as P on AB between A and B (NCASE=1 - Fig. 12a) and P on AB extended through A or B (NCASE=2 - Fig. 12b).

From these figures, illustrating the xy-cross section of the Angular Wedge, one can see that if the angular measure of the Wedge is given by $\theta = T-t$, then the length d of side AB is given by

$$d = \sqrt{R_1^2 + R_2^2 - 2R_1 R_2 \cos\theta} \tag{61}$$

By constructing perpendiculars from B to OA (possibly extended) and from A to OB (possibly extended), in all cases one obtains

$$\cos A = (R_1 - R_2 \cos\theta)/d \tag{62}$$

$$\cos B = (R_2 - R_1 \cos\theta)/d$$

3.5.3 Angular Wedge as Sum of Standard Wedges

This sub-section treats the case (Fig. 12a) in which the altitude OP lies <u>inside</u> the triangle AOB and divides side AB into segments AP and PB whose lengths are denoted by S_A and S_B respectively. The angle that PO makes with the positive x-axis (measured counterclockwise from the x-axis to the altitude) will be called α, and by considering triangles AOP and BOP one can easily obtain

$$\alpha = t + (\pi/2 - A) \tag{63a}$$
$$S_A = R_1 \cos A \tag{63b}$$
$$S_B = R_2 \cos B$$
$$r = R_1 \sin A \tag{63c}$$

Also, by definition of α, the point P will have x and y coordinates P_1 and P_2 given by

$$P_1 = r \cos\alpha \tag{64}$$
$$P_2 = r \sin\alpha$$

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Using the above relations, it remains to specify the Dimensional and Positional inputs describing the two sub-wedges and the relationship of their Input System to that assigned by the user (by choice of D, T, and t) to the Angular Wedge. This entails specifying Input Systems for each of the two sub-wedges in accordance with the rules governing the relationship between a Standard Wedge and its Input System (1.2.2). If these Standard Wedges were the six-face variety, this choice would consist merely of choosing the displacement D and deciding on the sense of the z-axis. Since they are, however, the special case of the five-face Wedges, in addition to these two decisions the directions of the x and z axes must be chosen because there is no require-ment relating to differentiating between the two vertices (away from the right angle) in this case except by fixing the directions of these two axes. Thus, there is a limited choice as to the Input Systems for these sub-wedges. For definiteness, the following convention for assigning Input Systems to these sub-wedges will be adhered to (this must be done in order to program the algorithm involved). The origin of the Input System for any of these sub-wedges will be at the point P, the z-axis of this system will be parallel to the altitude PO, the x-axis will be in the plane perpendicular to this altitude (positive sense in direction parallel to directed line segment from P to whichever of points A or B lies on the sub-wedge), and the y-axis chosen so as to form a right handed orthogonal coordinate system. Positioning the origin of the Input System of the Standard Wedges at the right angle of the object implies that D=0 has been assumed for the Standard Wedges. Furthermore, from Fig. 12a comparing it to the Standard Wedge and its Input Axes (Fig. 3) the length of either of these sub-wedges (which must be measured parallel to its Input System z-axis) will be r, the width (measured parallel to its y-axis) will be the length of the Angular Wedge, and the lower z height (measured parallel to the x-axis) will be either S_A or S_B respectively. (As mentioned above, the upper-z height is zero since the sub-wedges are not truncated.) These dimensions, together with the fact that both of these Wedges are assigned the same density as the Angular Wedge (because they are to be added) provide the Dimensional information for the sub-wedges considered as Standard Wedges. This description can be summarized (notation of 3.2.1 for Standard Wedges in parentheses on left side) as

$$
\begin{array}{lll}
\text{Displacement} & (D) & = 0 \\
\text{Length} & (H) & = r \\
\text{Width} & (W) & = H \text{ (Length of Angular Wedge} \quad (65) \\
\text{Lower-z height} & (R_1) & = S_A \text{ or } S_B \\
\text{Upper-z height} & (R_2) & = 0
\end{array}
$$

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and it is this information substituted into the equations derived in 3.2 for Standard Wedges which is used to provide the properties of these Standard Wedges with respect to their respective Input Systems.

Since the next step is to compute the properties of the combination of Standard Wedges with respect to the Input System of the Angular Wedge, it is necessary to describe the Input Systems of these two Standard Wedges with respect to that of the Angular Wedge. The relationships among these directions can be seen from Fig. 12a for this case. The notation used here is that $\vec{i}, \vec{j}, \vec{k}$ will be unit vectors in directions of the positive x,y and z axes of the Input System of a Standard Wedge and $\vec{i}', \vec{j}', \vec{k}'$ are unit vectors in directions of the x,y and z axes of the Input System of the Angular Wedge. Then, for the sub-wedge containing angle A, using the convention stated above, by observing Fig. 12a

$$\begin{aligned}
\vec{i} &= (\sin\alpha)\ \vec{i}' - (\cos\alpha)\ \vec{j}' \\
\vec{j} &= \vec{k}' \\
\vec{k} &= -(\cos\alpha)\ \vec{i}' - (\sin\alpha)\ \vec{j}'
\end{aligned} \tag{66}$$

Similarly, for the other sub-wedge containing angle B, its (Input) z-axis is the same, its x-axis is now directed in the opposite sense from the previous x-axis, and as a result its y-axis is also reversed in sense. This then can be summarized as

$$\begin{aligned}
\vec{i} &= -(\sin\alpha)\ \vec{i}' + (\cos\alpha)\ \vec{j}' \\
\vec{j} &= -\vec{k}' \\
\vec{k} &= -(\cos\alpha)\ \vec{i}' - (\sin\alpha)\ \vec{j}'
\end{aligned} \tag{67}$$

Equations (66, 67) provide the necessary information fo rotating the properties of the two sub-wedges from their respective Input Systems to a system with the same origin but parallel to the Input Axes of the Angular Wedge. To complete the description of the Input Systems of these two Standard Wedges requires the z-coordinate P_3 of $P=(P_1, P_2, P_3)$ the common origin of the Input Systems of both Standard Wedg (sub-wedges). From the overall Fig. 11 and the requirement that the Input xz-plane of the Standard Wedge bisect the Wedge, it is evident that P_3 must be given by

$$P_3 = D + W/2 \tag{68}$$

The coordinates of P provide the necessary information for translating the properties of the Angular Wedge (combination of the two sub-wedges) from a system with origin at P to the (parallel) Input System of the Angular Wedge.

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3.5.4 Angular Wedge as Difference of Standard Wedges

This sub-section treats the case (Fig. 12b, 12c) in which the altitude OP lies outside triangle AOB so that P will lie on BA extended through A or on BA extended through B, according as A is obtuse or B is obtuse respectively. The present case can be described by the condition that either A or B is obtuse, in contrast to the first case where both A and B were acute. Except for the change in signs in the rotation matrix, the difference between the two sub-cases, depending on whether it is A (case 2a) or whether it is B (case 2b) which is obtuse (equivalently whether it is A or B which is larger) is minor. Therefore, these two sub-cases are treated together and all that follows applies to both cases unless specifically stated otherwise, in which case two sets of equations are presented.

The length of the newly created line segment (AP or BP) will be called Q, otherwise the notation is as in the first case; Figures 12b and 12c illustrate the notation in the respective sub-cases. Equations (61), (62), and (63a, c) remain valid here also, but (63b) must be replaced by

$$Q = \begin{cases} -R_1 \cos A, & A > B \\ -R_2 \cos B, & B > A \end{cases} \tag{63b'}$$

as can be seen from the figures.

As before, it remains to specify the Dimensional and Positional inputs describing the two sub-wedges and the relationship of their Input Systems to that assigned by the user (by choice of D, T and t) to the Angular Wedge. The previously mentioned convention will be followed here also. Thus, from Fig. 11 comparing it to the Standard Wedge and its Input Axes (Fig. 3), one finds that again the length of either of these sub-wedges will be r, the width will be the length of the Angular Wedge, and the lower-z height will be either d+Q, for the larger wedge or Q, for the smaller; as before the upper-z height is zero. These dimensions, together with the fact that the larger of these two wedges is assigned the same density as the Angular Wedge and the smaller is assigned the negative of this density (so that they will be subtracted) provide the Dimensional information for the sub-wedges considered as Standard Wedges.

This description can be summarized (notation of 3.2.1 in parentheses on left side) as

Displacement   (D)   = 0
Length   (H)   = r
Width   (W)   = H (Length of the Angular Wedge
Lower-z height (R_1) = d+Q or Q   (6!
Upper-z   (R_2) = 0

and it is this information which is used to provide the properties of these Standard Wedges with respect to their common Input System. It is the previously assumed conventi which insures that the origin of the Input System of the two component Standard Wedges is located at P and that both Standard Wedges have the same Input System.

For case 2a, as in Fig. 12b, the common Input Axes of the two wedges are given in terms of the unit vectors ir the directions of the Input System of the Angular Wedge by (67) as for the "B Sub-Wedge" in case 1. For case 2b, as in Figure 12c, equation (66) holds as for the "A Sub-Wedge' in case 1. Thus the rotation matrices used to transform the properties of these sub-wedges from their Input Systems to systems parallel to the Input System of the Angular Wedç are all of the two forms

$$R = \begin{pmatrix} \mp \sin\alpha & 0 & -\cos\alpha \\ + \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \end{pmatrix}$$

As before, the coordinates of the common origin $P=(P_1, P_2, P_3)$ are given by equations (64) and (68).

Thus, all the equations have been derived for describi both the Dimensional and Positional information of the component sub-wedges into which any Angular Wedge can be modelled. This allows one to transform the properties of the component sub-wedges from their Input Systems to tha of the Angular Wedge, thus enabling one to obtain the prope of the Angular Wedge with respect to its own Input System.

3.6 CONCAVE PARTS

The purpose of this section is to show how one can treat a Concave Part as a combination of two Angular Wedges minus a Basic Part (Floating Sector). A Concave Part can be pictured as an Angular Wedge which has had a segment cut out of it by intersecting it with a right circular cylinder whose center is exterior to the Part and is chosen so as to form a non-convex object (see 1.2.4). Included herein are derivations of expressions for the dimensions of these three component Parts in terms of those of the overall Concave Part, and a relating of the position of the Input Systems of the components to the Input System of the Concave Part. (These results form the basis of the subroutine CONCAVE, described in 2.5.)

3.6.1 Notation for Dimensions of Concave Parts

This table contains the notation used for dimensions of Concave Parts in section 3.6. A Concave Part is illustrated in Fig. 5a; the FORTRAN symbols used in the figure are shown below in parentheses.

SYMBOL	EXPLANATION
D	Minimum z-coordinate of any point on the Concave Part, measured with respect to its own Input Axes (DZ).
H	Length of Concave Part, measured parallel to its Input z-axis (H).
t, T	Lower and upper angles of Concave Part (TH(1),TH(2)).
R_1, R_2	Radii at lower and upper angles of Concave Part (R1(1),R1(2))
x_c, y_c	X and Y (Input System) coordinates of the center of the exterior cylinder which defines the "concave area" of the Part (XC,YC).

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3.6.2 Modelling the Concave Part

The method of calculating the properties of the Concave Part requires that it be represented as a combination of Angular Wedges and Basic Parts, as described in 2.2, 2.4, 3.1 and 3.5. As in the case of the Angular Wedge, because the xy-cross section of a Concave Part is constant with respect to its own Input System, one can (without loss of generality) again derive expressions for relevant quantities using a two-dimensional approach confi to this plane. Thus (see Fig. 5b) it suffices to consider the Concave Part OP_1P_2 whose exterior center is located at point O', whose x and y coordinates with respect to the Input System of the Concave Part are (x_c, y_c). Using the notation of 3.6.1, the end points of the two straight sid of the Concave Part $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are give by

$$x_1 = R_1 \cos t$$
$$y_1 = R_1 \sin t$$
$$x_2 = R_2 \cos T \qquad ($$
$$y_2 = R_2 \sin T$$

These coordinates, with respect to a system parallel to th Input System but centered at (x_c, y_c), are given by

$$x_1' = x_1 - x_c$$
$$y_1' = y_1 - y_c$$
$$x_2' = x_2 - x_c \qquad ($$
$$y_2' = y_2 - y_c$$

Drawing (dotted) lines P_1O' and P_2O' defines angles Θ_3 and Θ_4 respectively, measured counterclockwise from the x-axis in the x'y'-system. These angles are given by

$$\Theta_3 = \tan^{-1} (y_1'/x_1') \qquad ($$

$$\Theta_4 = \tan^{-1} (y_2'/x_2')$$

The exterior radius RR may then be found from

$$RR = \sqrt{(x_1')^2 + (y_1')^2} \qquad ('$$

By drawing the (dotted) line P_1P_2 in Fig. 5b it can be seen that one can represent the Concave Part as the sum of Angular Wedges $O'P_1P_2$ and OP_1P_2 minus the Basic Part $O'P_1P_2$ whose third side is arc P_1P_2. This is essentially the requ decomposition of the Concave Part.

3.6.3 Components of the Concave Part

It remains to describe the Positional and Dimensional information for each of these three component Parts which when combined properly constitute the Concave Part. One can conveniently distinguish between the two Angular Wedges by referring to the first as the "Centered Angular Wedge", whose Input System is always chosen to coincide with that of the Concave Part, and the second as the "Exterior Angular Wedge" whose Input System is chosen parallel to the first but with origin located at the point (x_c, y_c, D) -these are coordinates in the Input System of the Concave Part. The Input System of the Basic Part is chosen to coincide with that of the Exterior Angular Wedge. Because all these Input Systems are parallel, no Rotational information is necessary; the angular displacements being taken care of by the angles T and t are considered as Dimensional data.

The three component Parts will be considered individually, starting with the Exterior Angular Wedge. From Figures 5a and 5b it can be seen that its length is equal to the length of the Concave Part, its lower and upper angles are equal to Θ_4 and Θ_3 respectively as given by (72), and both its radii are equal to RR as in (73). Since it is to be added, its density is chosen equal to that of the Concave Part. Finally, since its displacement is taken care of by a translation, it is set to zero for this Wedge. This description of the Exterior Angular Wedge can be summarized (notation of 3.5.1 for Angular Wedges in parentheses on the left) as

$$
\begin{aligned}
\text{Length} \quad &\text{(H)} = \text{H (Length of the Concave Part)} \\
\text{Lower angle} \quad &\text{(t)} = \Theta_4 \\
\text{Upper angle} \quad &\text{(T)} = \Theta_3 \\
\text{Displacement} \quad &\text{(D)} = 0 \\
\text{Radii} \quad &(R_1, R_2) = \text{RR}
\end{aligned}
\tag{74}
$$

For the Basic Part, the above description is valid. Its length equals that of the Concave Part, its angles are as above, its z-displacement is zero, and since it is a sector of a cylinder, its two end radii are identical and both equal to RR. Also, it is considered as a Floating Sector (NTYPE=1) but since it is to be subtracted, its density is chosen equal to the <u>negative</u> of that of the Concave Part.

Once the properties of these two Parts have been computed (by formulae in 3.1 and 3.5) they can be added to obtain the properties of the combination of these two Parts (actually an object with a negative mass, i.e., to be deleted), which using the formulas in 3.3 can be transformed to the Input System of the Concave Part via a translation from $(x_c, y_c, 1)$.

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For the Centered Angular Wedge, its description is similar to that of the Concave Part itself; its length, its angles, its displacement and its radii are identical to that of the overall Concave Part. The properties of this component can be found using the methods of 3.5, and can be added to those of the combination of the first two components to obtain the properties of the overall Concave Part with respect to its Input System.

$x^2 + y^2 = \{R1(1)\}^2$

$x^2 + y^2 = \{R2(1)\}^2$
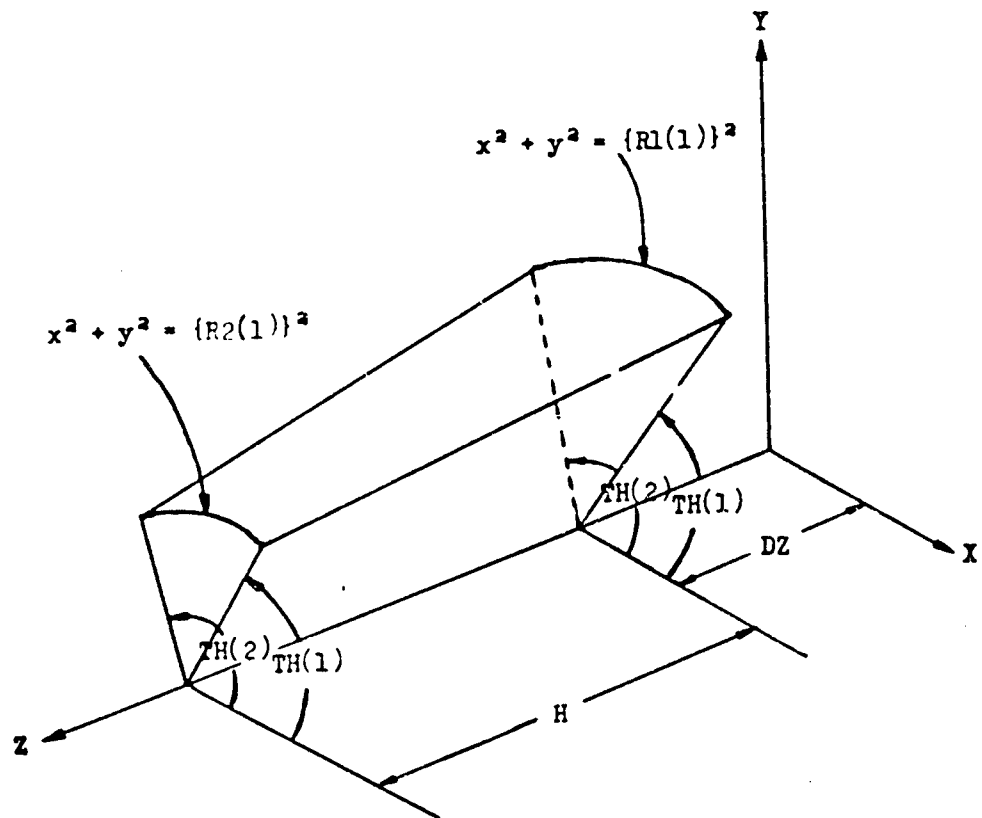
Length: H   Displacement: DZ

Radii: R1(1),R2(1)   Angles: TH(1),TH(2)

Fig. 1   Prototype Sector

109
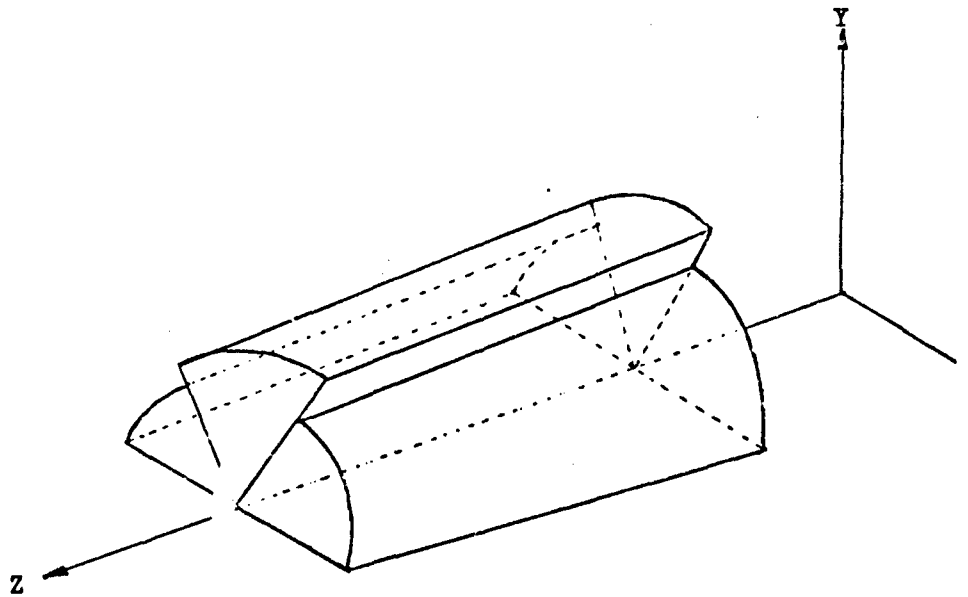
Fig. 2   Upper Half of Basic Part
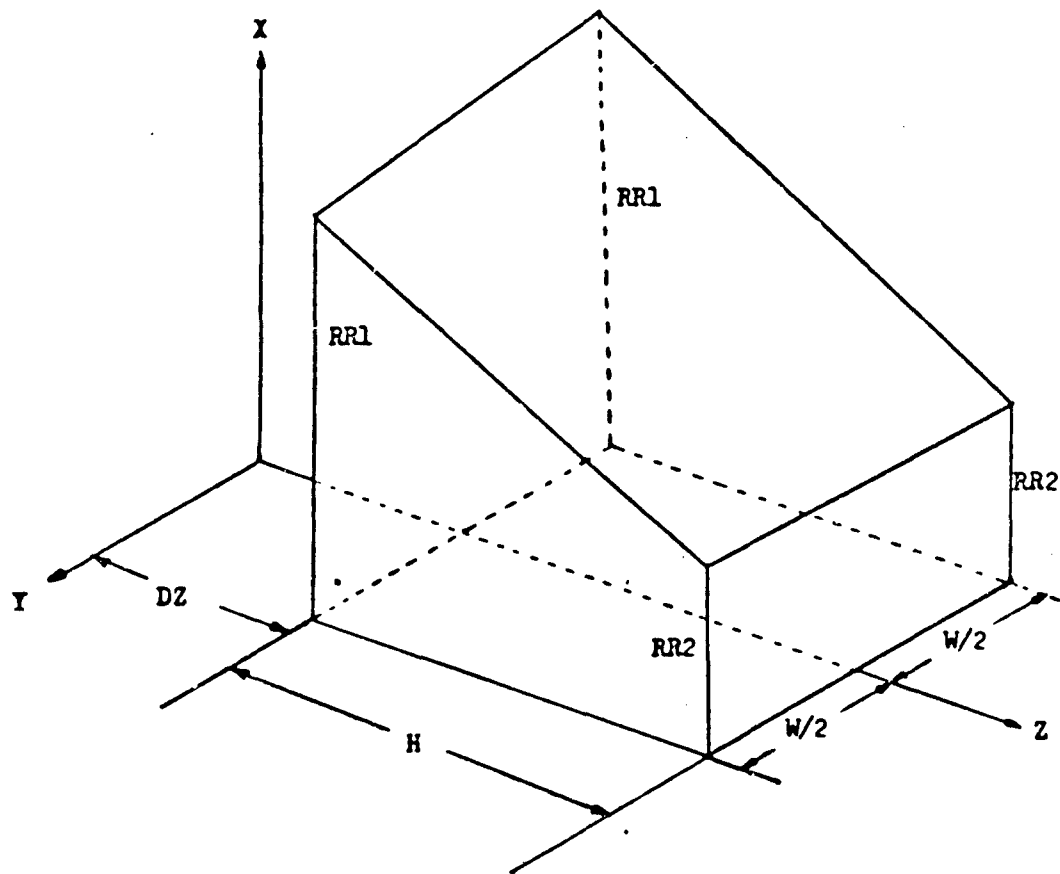
Length: H   Displacement: DZ

Width: W   Heights: RR1,RR2

Fig. 3a   Standard Wedge

111
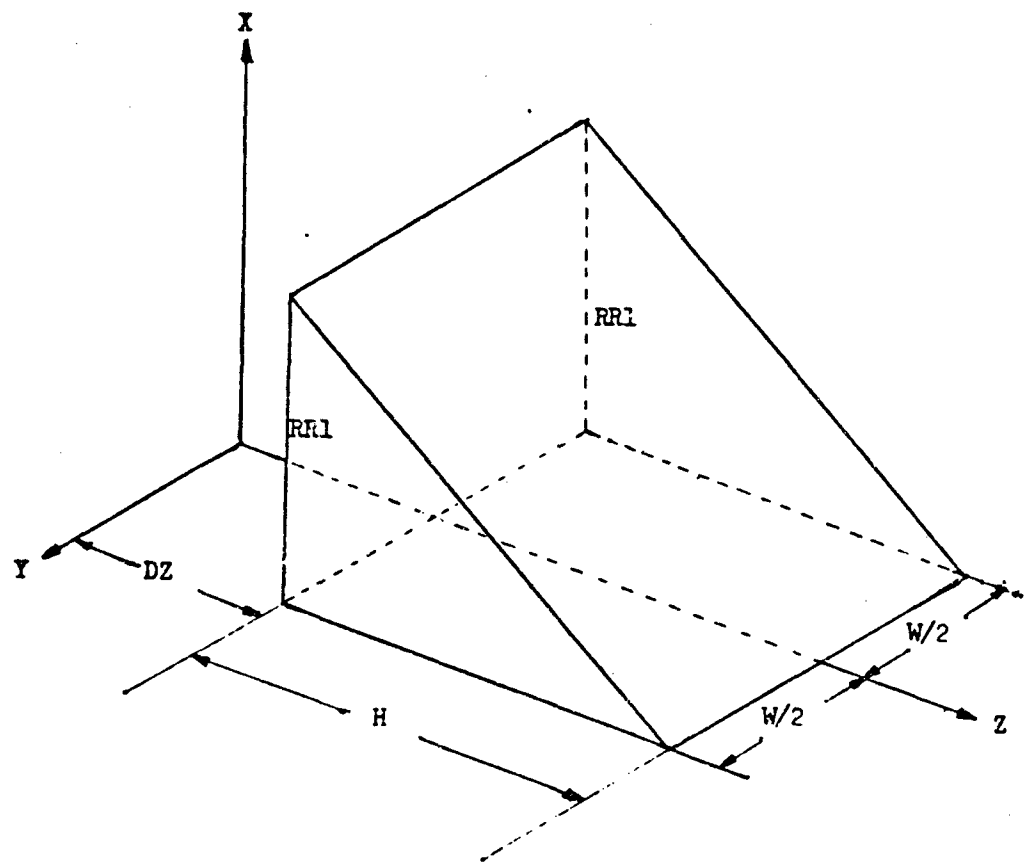
Length: H   Displacement: DZ

Width: W   Heights: RR1; RR2=0

Fig. 3b   Standard Wedge, RR2=0

112
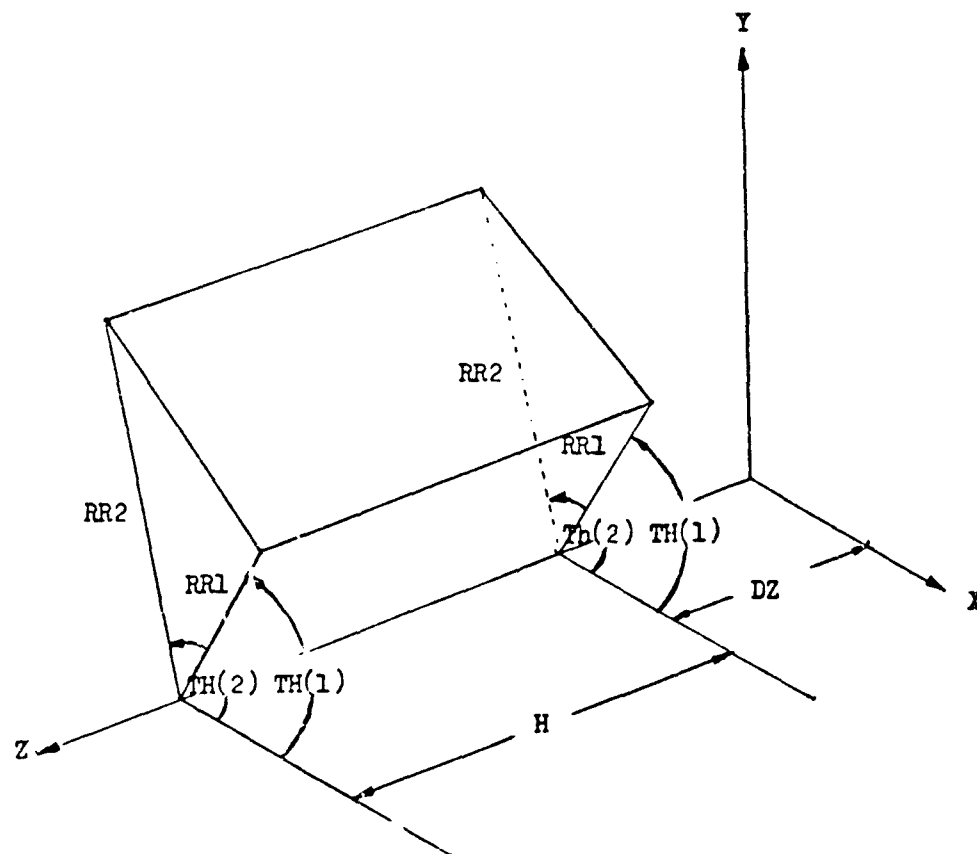
Length: H   Displacement: DZ

Radii: RR1,RR2   Angles: TH(1),TH(2)

Fig. 4   Angular Wedge

113
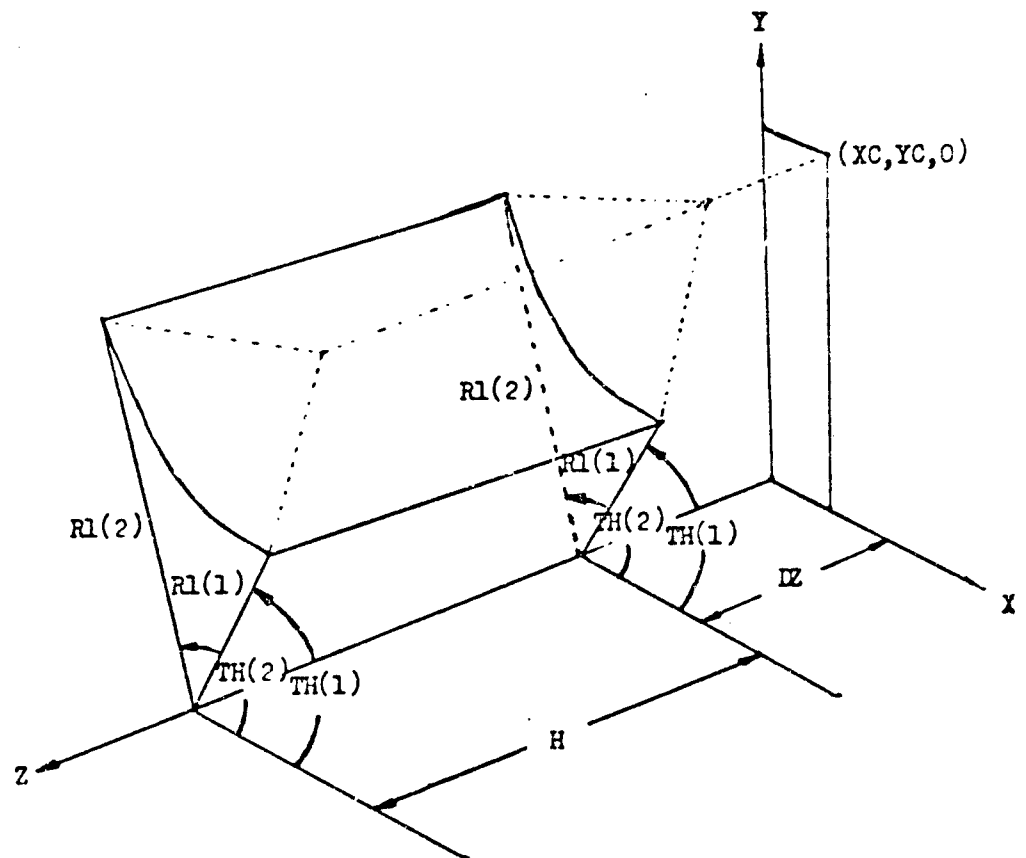
Length: H   Displacement: DZ

Radii: R1(1),R1(2)   Angles: TH(1),TH(2)

Exterior center for cylinder on line thru (XC,YC,0)

Fig. 5a   Concave Part

114
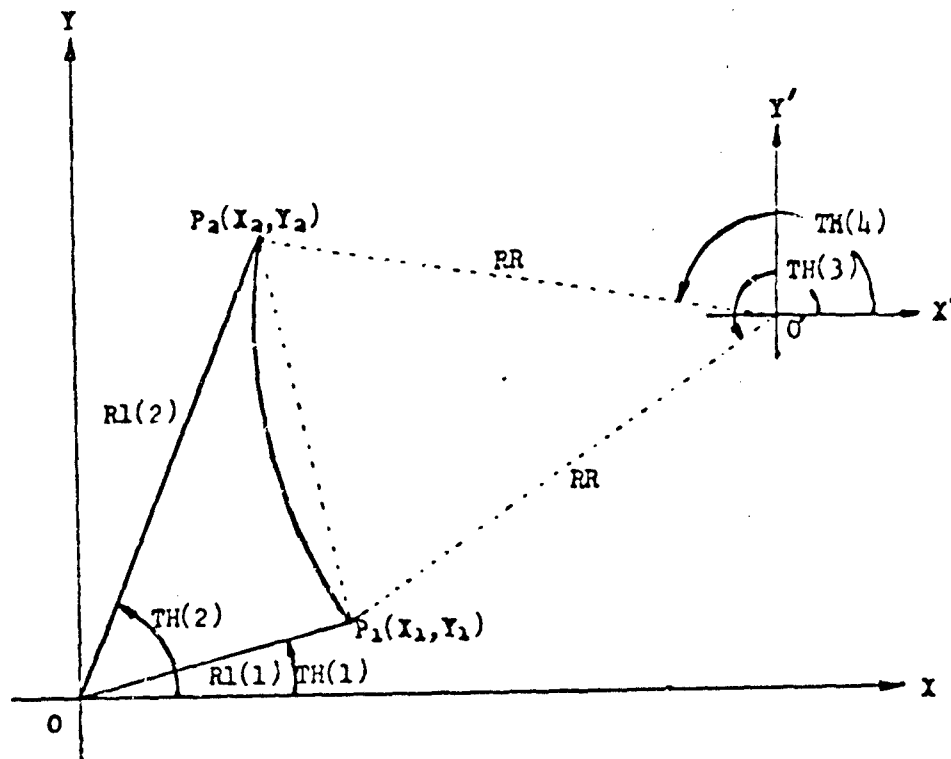
Lower Radius: Rl(1)   Lower Angle: TH(1)

Upper Radius: Rl(2)   Upper Angle: TH(2)

Exterior center O′ at (XC,YC)

Fig. 5b   Cross Section of Concave Part

115
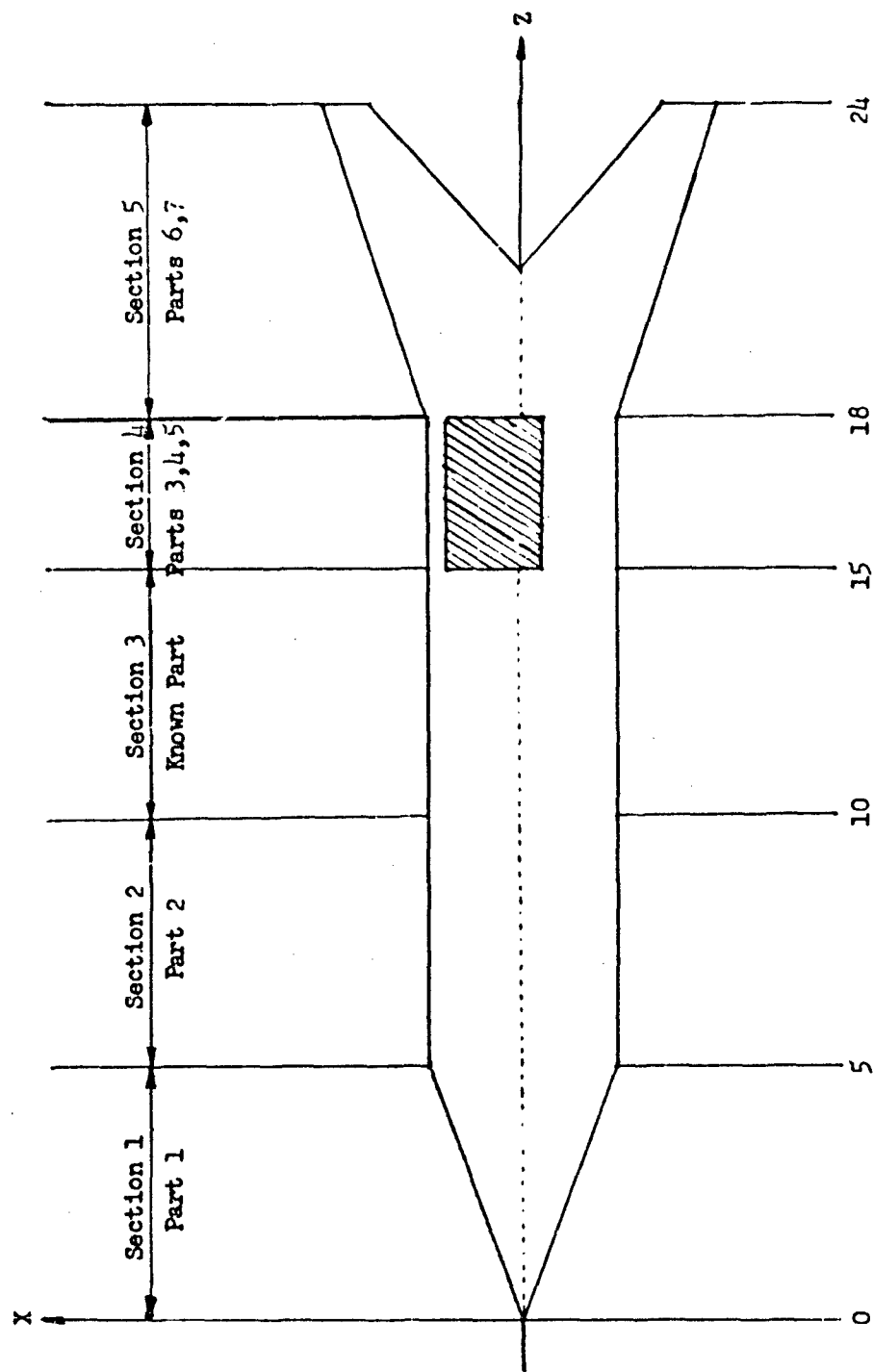
Fig. 6 Sample Body - Sample Case 1

116
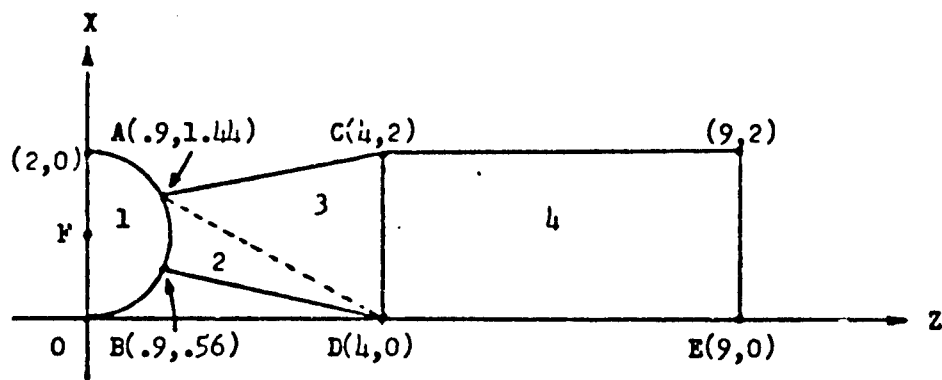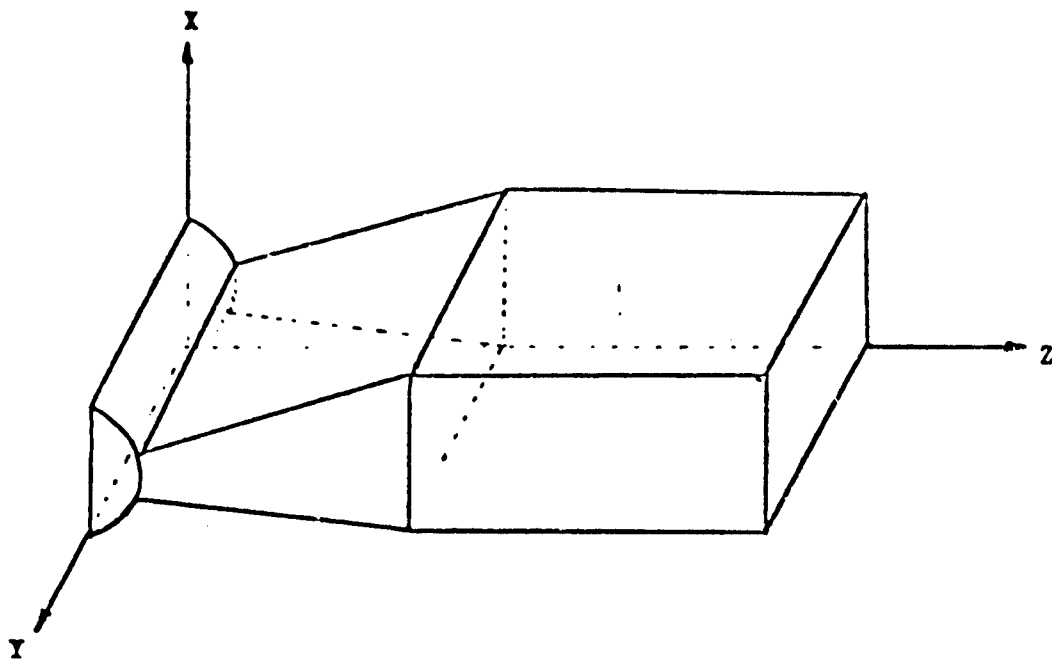
Fig. 7   Sample Body - Sample Case 2

117
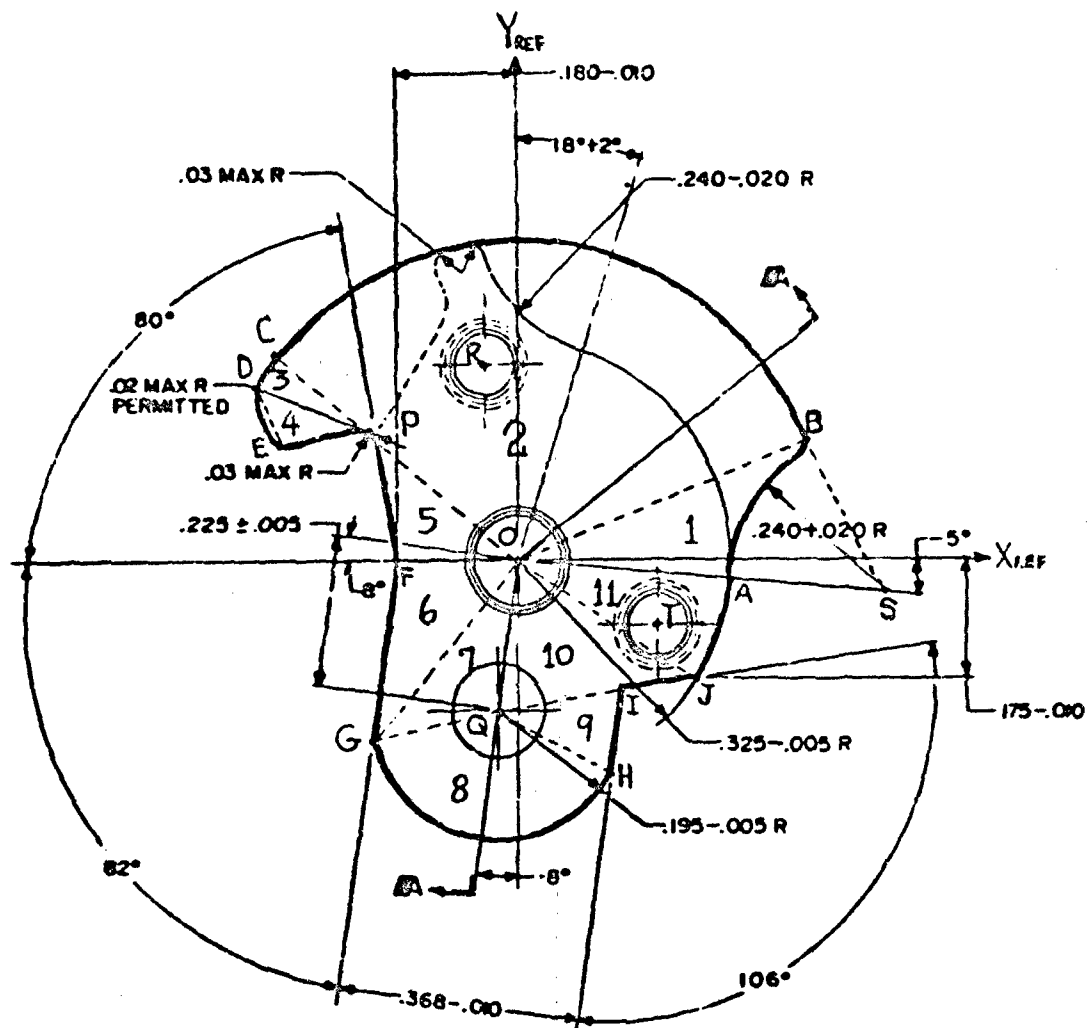
NOTE:

1. INTERPRET DRAWING IN ACCORDANCE WITH STD MIL-A--2550 AND ALL DOCUMENTS CONTAINED THEREIN.

2. MATERIAL: SINTERED BRASS, CLASS B, ASTM B282. ALTERNATIVE MATERIAL: BRASS, HALF HARD SPEC ASTM—B16.

3. 125/ALL OVER EXCEPT AS NOTED.

Fig. 8a   View A

118

Fig. 8b   View B

119

.220-.006

.005 +.015 x 45° .005 +.005 x 45°

.131+.003 DIA

.145+.005 DIA .230+.010

.152+.005 DIA

Fig. 9a   SECTION A-A

.120+.010 DIA

.0932 +.0015 DIA
2 HOLES

.021+.010

.005 MAX x 45°

.005 MAX x 45°

Fig. 9b   SECTION B-B

120
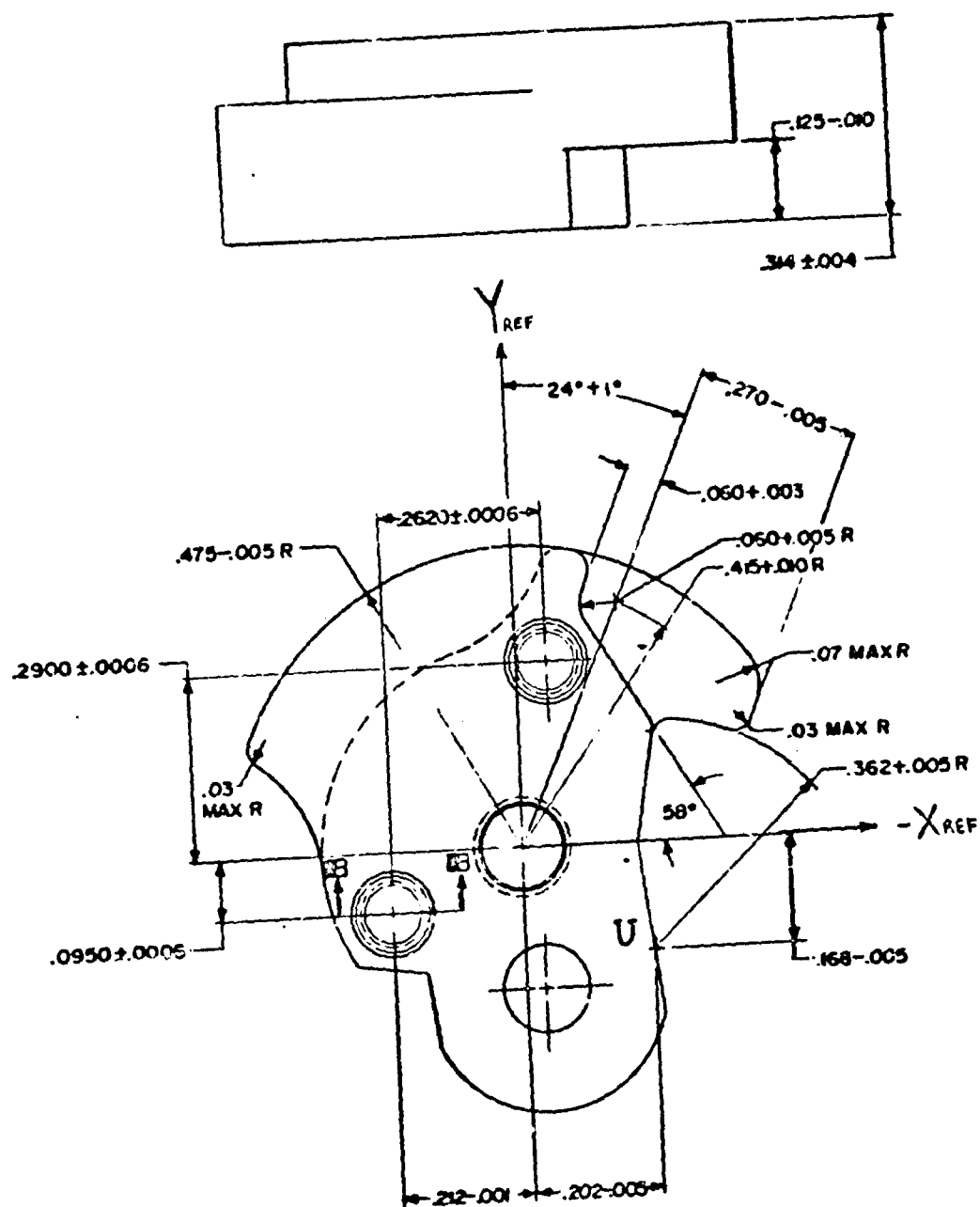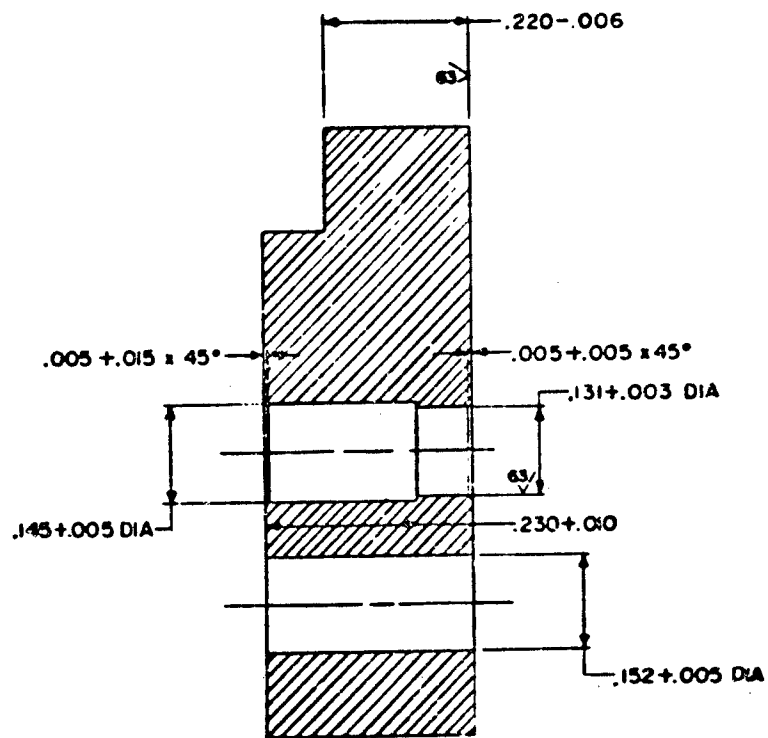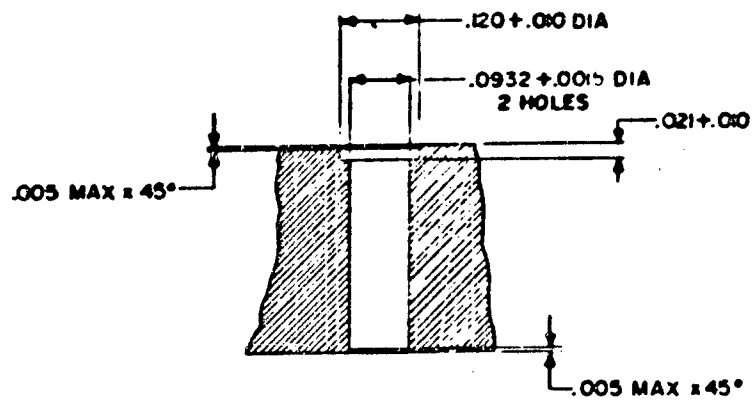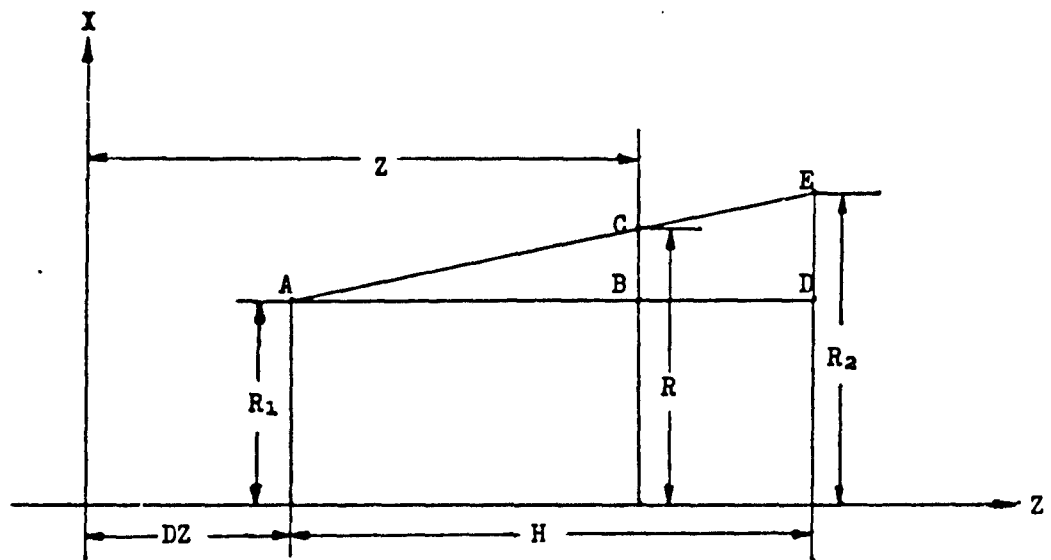
Fig. 10   Side View of Prototype Sector

121
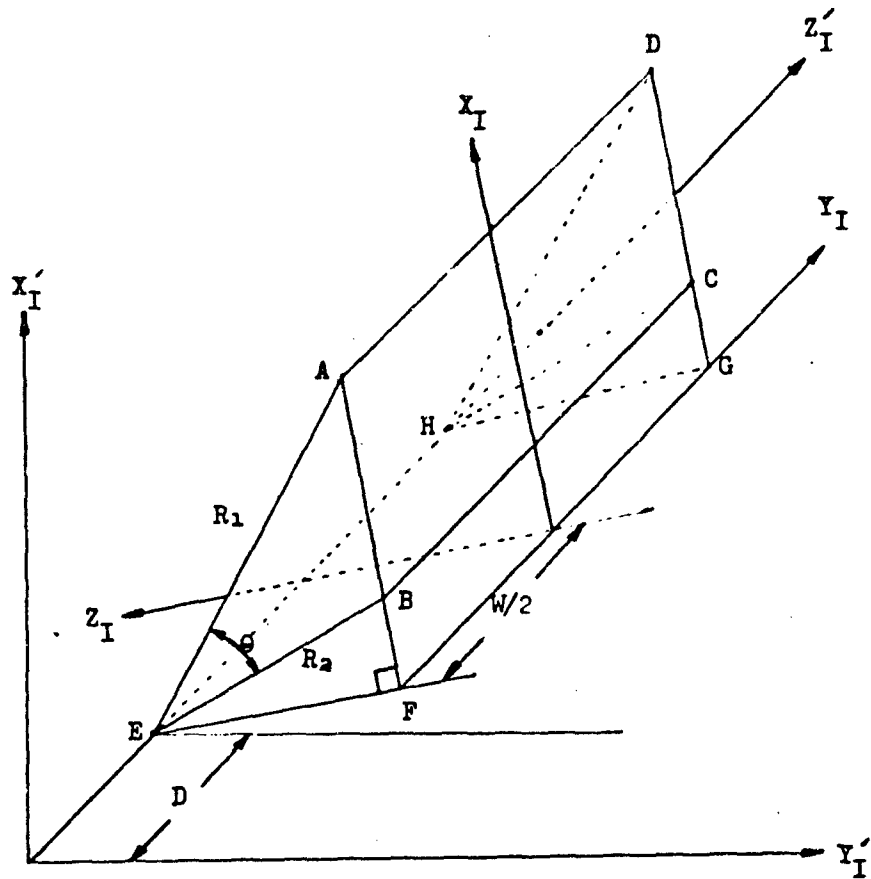
Input Axes of Right Angle Wedges: X_I, Y_I, Z_I

Input Axes of Angular Wedge: X_I', Y_I', Z_I'

Radii of Angular Wedge: R_1, R_2

Angle Measure of Angular Wedge: θ

Fig. 11   Angular Wedge, Components and Input Systems

122
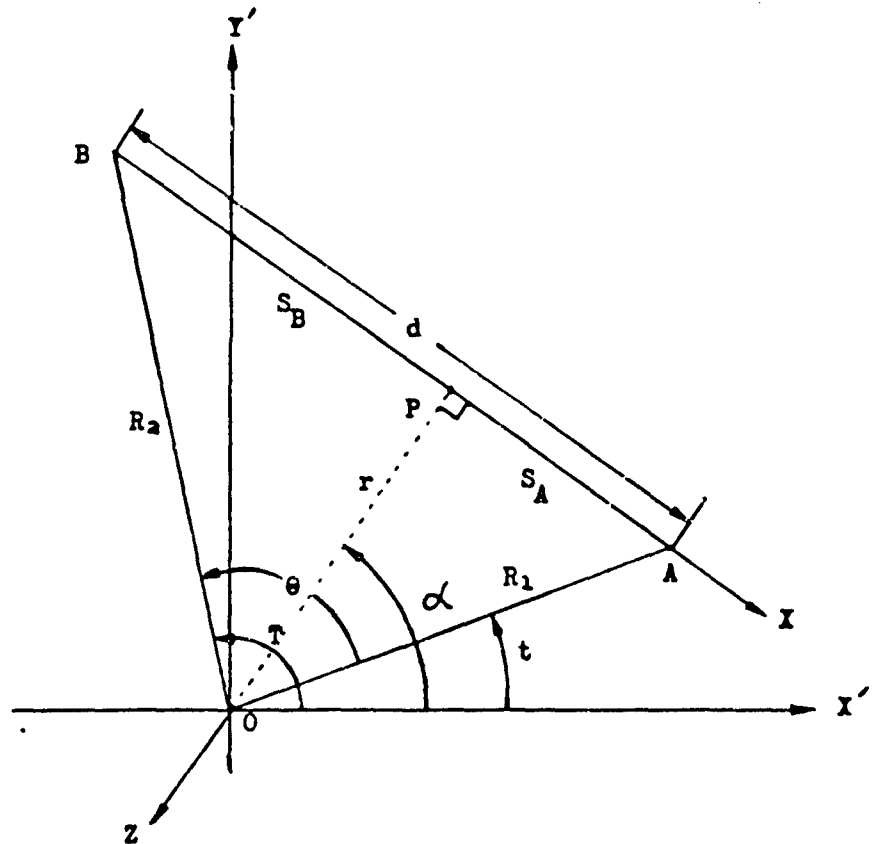
Angular Wedge Cross Section: AOB

Input Axes for Component Sub-Wedge AOP: X,Y,Z

Input Axes for Angular Wedge: X',Y',Z'

Angular Wedge Angles: t,T   ($\theta = T - t$)

Angular Wedge Radii: R_1,R_2

Perpendicular OP lies at angle α

Fig. 12a   Cross Section of Angular Wedge - Case 1
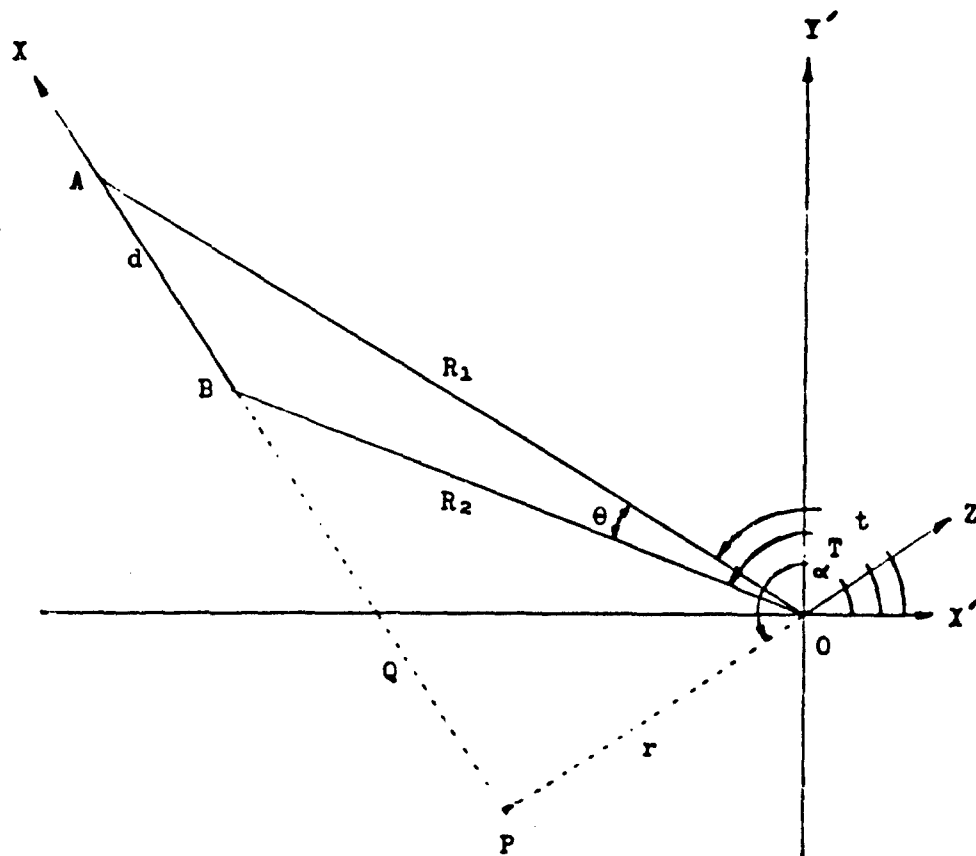
123

Angular Wedge Cross Section: AOB

Input Axes for Component Sub-Wedges: X,Y,Z

Input Axes for Angular Wedge: X,Y,Z

Angular Wedge Angles: t,T   ($\theta = T - t$)

Angular Wedge Radii: R_1,R_2

Perpendicular OP lies at angle α

Fig. 12b   Cross Section of Angular Wedge - Case 2a

124

Angular Wedge Cross Section: AOB

Input Axes for Component Sub-Wedges: X, Y, Z

Input Axes for Angular Wedge: X, Y, Z

Angular Wedge Angles: t, T $(\theta = T - t)$

Angular Wedge Radii: R_1, R_2

Perpendicular OP lies at angle α

Fig. 12c   Cross Section of Angular Wedge - Case 2b

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REFERENCES

1. Isaacson, E. and Keller, H.B., _Analysis of Numerical Methods_, John Wiley & Sons Inc., 1966.

2. Lacher, E. "MOMENTS, A Computer Program to Calculate Moments and Products of Inertia of Asymmetric Shells and Other Bodies," Picatinny Arsenal Technical Report 4143, July 1971

3. Leiber, L.R. and Leiber H.G., "_The Einstein Theory of Relativity_," Holt, Reinhart and Winston, 1945

4. Ralston, A., _A First Course in Numerical Analysis_, McGraw-Hill, Inc., 1965

5. Symon, K.R., _Mechanics_ (second edition), Addison-Wesley Publishing Company Inc., 1960

APPENDIX A - Sample Case 1

This Appendix contains a listing of the input cards,
followed by the output from MOMENTS-II for Sample Case 1
as described in 1.5.1.

For the data cards, the numbers in the far left
column are card numbers; they are listed here for easier
reference and should not be considered part of the actual
data punched on the input cards which begins to the right
of the heavy line.

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```
** MOMENTS-VI **     SAMPLE CASE 1
 1          .2750      1      1      0      1
 2     NOEF      0.0   5.0    0.0    0.0    0
 3               2.0
 4
 5   FRONT PART   5.0   5.0    0.0    5.0    0
 6               2.0
 7   BASIC        2.0   2.0    0.0   15.0    0
 8               2.0
 9
10   DELETION     3.0   3.0   -0.0   15.0    0
11               1.0
12               1.0
13   INSERTION    0.5000 0.0    0.0   15.0    0
14               3.0    0.5
15               1.0
16               1.0
17   TAIL         0.5000 0.0    0.0   18.0    0
18               6.0    0.2
19               2.0
20               4.0
21   TAIL DLT.    0.0    3.0   -0.0   21.0    0
22               0.0
23               3.0
24               62.9319 17.2788 0.0    0.0    0.0
25               53.2762  0.0   53.2762 0.0    0.0
26                                    12.5 KNOWN PART
27                                     0.0  74.5575
```

** MOMENTS-II **   SAMPLE CASE 1

M O M E N T S - I I   O P T I O N S

DEFAULT DENSITY =   .2750

DEFAULT NO. OF SECTORS/PART =   1

OUTPUT TYPE = PARTIAL

NO. OF BASIC PARTS =   7

NO. OF WEDGES =   0

NO. OF CONCAVE PARTS =   0

NO. OF KNOWN PARTS =   1

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** MOMENTS-II **   SAMPLE CASE 1

INPUT DATA - BASIC PARTS

NO.	IDENTIFICATION	LENGTH	DENSITY	DZ(INP. SYS.)	NTYPE-SECT.	R1(1)	R2(1)	TH(1)	TH(2)
1	NOSE	5.0000	.27500	6.0000	0	1	0.0000	2.0000	.	.
2	FRONT PART	5.0000	.27500	5.0000	0	1	2.0000	2.0000	.	.
3	BASIC	3.0000	.30000	15.0000	0	1	2.0000	2.0000	.	.
4	DELETION	3.0000	-.30000	15.0000	0	1	1.0000	1.0000	.	.
	INPUTS RELATIVE TO AXES WITH ORIGIN AT (			.5000,	0.0000,	0.0000)				
5	INSERTION	3.0000	.50000	15.0000	0	1	1.0000	1.0000	.	.
	INPUTS RELATIVE TO AXES WITH ORIGIN AT (			.5000,	0.0000,	0.0000)				
6	TAIL	6.0000	.20000	18.0000	0	1	2.0000	4.0000	.	.
7	TAIL DLT.	3.0000	-.20000	21.0000	0	1	0.0000	3.0000	.	.

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** MOMENTS-II ** SAMPLE CASE 1

I N P U T   D A T A   -   K N O W N   P A R T S

NO.	IDENTIFICATION	MASS	VOLUME	C.G.(W.R.T. REF. AXES)			MOMENTS AND PRODUCTS OF INERTIA (W.R.T. INPUT AXES)					
							I(XX)	I(XY)	I(YY)	I(XZ)	I(YZ)	I(ZZ)
1	KNOWN PART	17.2784	62.8319	0.0000	0.0000	12.5000	53.2762	0.0000	53.2762	0.0000	0.0000	34.5575

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PROPERTIES OF BASIC PARTS

NO.	IDENTIFICATION	VOLUME	MASS	C.G.(W.R.T. RFF. AXES)		
1	NOSE	20.9440	5.7596	(0.0000,	0.0000,	3.7500)
2	FRONT PART	62.8319	17.2788	(0.0000,	0.0000,	7.5000)
3	BASIC	37.6991	11.3097	(0.0000,	0.0000,	16.5000)
4	DELETION	-9.4248	-2.8274	(.5000,	0.0000,	16.5000)
5	INSERTION	9.4248	4.7124	(.5000,	0.0000,	16.5000)
6	TAIL	175.9292	35.1858	(0.0000,	0.0000,	21.6429)
7	TAIL DLT.	-28.2743	-5.6549	(0.0000,	0.0000,	23.2500)

** MOMENTS-II **   SAMPLE CASE 1

P R O P E R T I E S   O F   K N O W N   P A R T S

NO.	IDENTIFICATION	MOMENTS AND PRODUCTS OF INERTIA (W.R.T. REF. AXES)					
		I(XX)	I(YY)	I(XY)	I(XZ)	I(YZ)	I(ZZ)
1	KNOWN PART	2753.0887	2753.0887	0.0000	0.0000	0.0000	34.5572

** MOMENTS-II ** SAMPLE CASE 1

P R O P E R T I E S   O F   E N T I R E   B O D Y

MASS =   83.0428

NET VOLUME =   331.9617

POSITIVE VOLUME =   369.6608

NEGATIVE VOLUME =   -37.6991

COORDINATES OF CG =(.0113,   0.0000,   14.6302)

COMPONENTS OF INERTIA TENSOR WITH RESPECT TO REFERENCE AXES

I(XX) =   21084.7635
I(XY) =   0.0000
I(YY) =   21085.2348
I(XZ) =   -15.5509
I(YZ) =   0.0000
I(ZZ) =   271.7792

COMPONENTS OF INERTIA TENSOR WITH RESPECT TO C.G. AXES

I(XX) =   3310.0510
I(XY) =   0.0000
I(YY) =   3310.5115
I(XZ) =   -1.7622
I(YZ) =   0.0000
I(ZZ) =   271.7685

PRINCIPAL MOMENTS   PRINCIPAL AXES

3310.0520   (.9999998, 0.0000000, -.0005800)

3310.5115   (0.0000000, 1.0000000, 0.0000000)

271.7674   (.0005800, 0.0000000,   .9999998)

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APPENDIX B - Sample Case 2

This Appendix contains a listing of the input cards, followed by the output from MOMENTS-II for Sample Case 2 as described in 1.5.2.

For the data cards, the numbers in the far left column are card numbers; they are listed here for easier reference and should not be considered part of the actual data punched on the input cards which begins to the right of the heavy line.

```
 1
 2    ** MOMENTS-II **   SAMPLE CASE 2
 3   PART 1  BP    0.2    3.0          2    1    1    270.0    90.0    1    1
                                       0.0                    0.0         1    1
 4           1.0
 5           1.0
 6    0.0    0.0    0.0    1.0    0.0    0.0    0.0    1.0    0.0
 7           1.0    0.0    0.0
 8   PART 3  AW    3.0          2.0    2.0    3.4164    90.0    155.147    1    1
                                       0.0              0.0                1    1
 9    0.0    0.0    1.0    1.0    0.0    0.0
10    0.0    0.0    4.0
11   PART 4  SW    5.0          2.0    2.0    2.0    3.0    0    1
12    0.0    1.5    4.0
13   PART 2  CP    3.0          3.4164    0.0    3.1509    155.147    169.687    1    1
                                          0.0                          0.0       1    1
14   -4.0    1.0    1.0
15    0.0    0.0    1.0    0.0    0.0    1.0    0.0
16    0.0    0.0    4.0    0.0    0.0
```

M O M E N T S - I I   O P T I O N S

DEFAULT DENSITY =   .2000

DEFAULT NO. OF SECTORS/PART = 1

OUTPUT TYPE = PARTIAL

NO. OF BASIC PARTS = 1

NO. OF WEDGES = 7

NO. OF CONCAVE PARTS = 1

NO. OF KNOWN PARTS = 0

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I N P U T   D A T A   -   B A S I C   P A R T S

NO.	IDENTIFICATION	LENGTH	DENSITY	DZ(INP. SYS.)	NTYPE-SECT.	RI(I)	R2(I)	TH(1)	TH(2)
1	PART 1 BP	3.0000	.20000	0.0000	1 1	1.0000	1.0000	270.000	90.000

INPUTS RELATIVE TO AXES (0.0000, 0.0000, 1.0000), (1.0000, 0.0000, 0.0000), (0.0000, 1.0000, 0.0000)
INPUTS RELATIVE TO AXES WITH ORIGIN AT (1.0000, 0.0000, 0.0000)

I N P U T   D A T A   -   W E D G E S

NO.	IDENTIFICATION	LENGTH	DENSITY	DZ(INP. SYS.)	MTYPE	RR1	RR2	WIDTH	TH(1)	TH(2)
1	PART 3 AW	3.0000	.20000	0.0000	ANG	2.0000	3.4164		90.000	155.1 7

INPUTS RELATIVE TO AXES (0.0000, 0.0000, 1.0000), (1.0000, 0.0000, 0.0000), (0.0000, 1.0000, 0.0000)
INPUTS RELATIVE TO AXES WITH ORIGIN AT (0.0000, 0.0000, 4.0000)

NO.	IDENTIFICATION	LENGTH	DENSITY	DZ(INP. SYS.)	MTYPE	RR1	RR2	WIDTH	TH(1)	TH(2)
2	PART 4 SW	5.0000	.20000	0.0000	STD	2.0000	2.0000	3.0000		

INPUTS RELATIVE TO AXES WITH ORIGIN AT (0.0000, 1.5000, 4.0000)

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I N P U T   D A T A - C O N C A V E   P A R T S

NO.	IDENTIFICATION	LENGTH	DENSITY	DZ(INP. SYS.)	R1(1)	R1(2)	TH(1)	TH(2)	X-CENTER	Y-CENTER
1	PART 2 CP	3.0000	.20000	0.0000	3.4164	3.1509	155.147	169.687	-4.0000	1.0000

INPUTS RELATIVE TO AXES   (0.0000, 0.0000, 1.0000), (1.0000, 0.0000, 0.0000), (0.0000, 1.0000, 0.0000)
INPUTS RELATIVE TO AXES WITH ORIGIN AT (0.0000, 0.0000, 4.0000)

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** MOMENTS-II **   SAMPLE CASE 2

PROPERTIES OF BASIC PARTS

NO.	IDENTIFICATION	VOLUME	MASS	C.G.(W.R.T. REF. AXES)		
1	PART 1 BP	4.7124	.9425	(1.0000,	1.5000,	.4244)

** MOMENTS-II **   SAMPLE CASE 2

P R O P E R T I E S   O F   W E D G E S

NO.	IDENTIFICATION	VOLUME	MASS	C.G.(W.R.T. REF. AXES)		
1	PART 3 AW	9.3000	1.8600	(1.1453,	1.5000,	2.9667)
2	PART 4 SW	30.0000	6.0000	(1.0000,	1.5000,	6.5000)

** MOMENTS-II ** SAMPLE CASE 2

P R O P E R T I E S   O F   C O N C A V E   P A R T S

NO.	IDENTIFICATION	VOLUME	MASS		C.G.(d.R.T. REF. Axes)	
1	PART 2 CP	3.8776	.7755	(	.6515, 1.5000, .07165)

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P R O P E R T I E S   O F   E N T I R E   B O D Y

MASS =   9.5780

NET VOLUME =   47.8900

POSITIVE VOLUME =   47.8900

NEGATIVE VOLUME =   0.0000

COORDINATES OF CG =(1.0000,   1.5000,   4.8499)

COMPONENTS OF INERTIA TENSOR WITH RESPECT TO REFERENCE AXES

I(XX) =   315.7649
I(XY) =   -14.3670
I(YY) =   299.3728
I(XZ) =   -46.4523
I(YZ) =   -69.6785
I(ZZ) =   41.0760

COMPONENTS OF INERTIA TENSOR WITH RESPECT TO C.G. AXES

I(XX) =   68.9252
I(XY) =   -.0000
I(YY) =   64.5057
I(XZ) =   -.0000
I(YZ) =   -.0000
I(ZZ) =   9.9475

PRINCIPAL MOMENTS   PRINCIPAL AXES

68.9252   (1.0000000, 0.0000000, -.0000006)

64.5057   (0.0000000, 1.0000000, 0.0000000)

9.9475   (.0000006, 0.0000000, 1.0000000)

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APPENDIX C - Sample Case 3

This Appendix contains a listing of the input cards, followed by the output from MOMENTS-II for Sample Case 3 as described in 1.5.3.

For the data cards, the numbers in the far left column are card numbers; they are listed here for easier reference and should <u>not</u> be considered part of the actual data punched on the input cards which begins to the right of the heavy line.

```
** MOMENTS-IT **      SAMPLE CASE 3

NO.  2   .308    1    .005    2    0        19.8    137.86      1
     L2  .475
         .475
NO.  3        L2         .005          .094    137.86  164.414   1
              .475
         -.9246
         -.9246
NO.  8  -.24171  .193793 .005   0.0   195.922  337.249  1
         .195
         .195
NO. 11  -.2229l     L2   .005   0.0   327.421  355.0    1
         .325
         .325
HOL= 1          .005   -.309   .094    1                 1
        -.0655
        -.0655
HDCH 2  -.0466      .005   -.309   .094    0              1
        -.0466
HOL= 3  -.0466      .005   -.309   .094    0
         .0466
         .0466
    27   .212
HOL= 4   .076       .005   -.309   .094    0              1
         .076
NO. 31  -.031314    -.2229l   0.0   .288834   .18        1
NO.  5      L2      .005     .094     .18    137.86  180.0    1
NO.  6      L2      .005     .094   .352463  180.0  231.621
NO.  7      L2      .005     .094    .225    231.621 262.0
NO.  9      L2      .005     .094   .352463  .195    262.0   8.904   1
NO. 35  -.031314    -.2229l   0.0    .195   197089  337.249
NO. 17      L2      .005      .225    262.0  327.421   1
NO.  1      L2      .005      .325    .475   355.0    19.8
        -.040243
NO.  4      L2      .005    .10052  .198748  294.0  344.414   1
```

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** MOMENTS-II ** SAMPLE CASE 3

M O M E N T S - I I O P T I O N S

DEFAULT DENSITY = .3080

DEFAULT NO. OF SECTORS/PART = 1

OUTPUT TYPE = PARTIAL

NO. OF BASIC PARTS = 8

NO. OF WEDGES = 5

NO. OF CONCAVE PARTS = 2

NO. OF KNOWN PARTS = 0

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** MOMENTS-II **   SAMPLE CASE 3

INPUT DATA - BASIC PARTS

NO.	IDENTIFICATION	LENGTH	DENSITY	DZ(INP. SYS.)	NTYPE-SECT.	R1(1)	R2(1)	TM(1)	TM(2)
1	NO. 2 L2	.0950	.30800	.0940	1	.4750	.4750	19.800	137.060
2	NO. 3 L2	.0950	.30800	.0940	1	.1925	.1925	137.860	164.214
	INPUTS RELATIVE TO AXES WITH ORIGIN AT (-.2142, .1938, 0.0000)								
3	NO. 8 L2	.0950	.30800	.0940	1	.1950	.1950	195.922	337.349
	INPUTS RELATIVE TO AXES WITH ORIGIN AT (-.0313, -.2228, 0.0000)								
4	NO. 11 L2	.0950	.30800	.0940	1	.3250	.3250	327.421	355.000
5	HOLE 1	.0950	-.30800	.0940	0	.0655	.0655	.	.
6	HOLE 2	.0950	-.30800	.0940	0	.0466	.0466	.	.
	INPUTS RELATIVE TO AXES WITH ORIGIN AT (-.0500, .2900, 0.0000)								
7	HOLE 3	.0950	-.30800	.0940	0	.0466	.0466	.	.
	INPUTS RELATIVE TO AXES WITH ORIGIN AT (.2120, -.0950, 0.0000)								
8	HOLE 4	.0950	-.30800	.0940	0	.0760	.0760	.	.
	INPUTS RELATIVE TO AXES WITH ORIGIN AT (-.0313, -.2228, 0.0000)								

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** MOMENTS--II **   SAMPLE CASE 3

INPUT DATA - WEDGES

NO.	IDENTIFICATION	LENGTH	DENSITY	DZ(INP. SYS.)	WTYPE	RR1	RR2	WIDTH	TH(1)	TH(2)
1	NO. 5 L2	.0950	.30800	.0940	ANG	.2888	.1800	*	137.860	180.010
2	NO. 6 L2	.0956	.30800	.0940	ANG	.1800	.3525	*	180.000	231.621
3	NO. 7 L2	.0950	.30800	.0940	ANG	.3525	.2250	*	231.621	262.010
4	NO. 9 L2	.0950	.30800	.0940	ANG -.0313,	.1950 -.2228,	.1971 0.0000)	*	337.049	8.944
			INPUTS RELATIVE TO AXES WITH ORIGIN AT (							
5	NO. 10 L2	.0950	.30800	.0940	ANG	.2250	.3250	*	262.000	327.421

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** MOMENTS-II **   SAMPLE CASE 3

INPUT DATA - CONCAVE PARTS

NO.	IDENTIFICATION	LENGTH	DENSITY	DZ(INP. SYS.)	RI(1)	RI(2)	TH(1)	TH(2)	X-CENTER	Y-CENTER
1	NO. 1 L?	.0950	.30800	.0940	.3250	.4750	355.000	19.800	.5629	-.0492
2	NO. 4 L2	.0950	.30800	.0940	.1005	.1987	294.000	344.414	.2036	-.4152
		INPUTS RELATIVE TO AXES WITH ORIGIN AT (			-.4056,	.2472,	0.0000)			

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** MOMENTS-II **   SAMPLE CASE 3

P R O P E R T I E S   O F   B A S I C   P A R T S

NO.	IDENTIFICATION	VOLUME	MASS	C.G.(W.R.T. REF. AXES)		
1	NO. 2 L2	.0221	.0068	(.0511,	.2586,	.1415)
2	NO. 3 L2	.0008	.0003	(-.3255,	.2552,	.1415)
3	NO. 8 L2	.0045	.0014	(-.0372,	-.3221,	.1415)
4	NO. 11 L2	.0024	.0007	(.2031,	-.0691,	.1415)
5	HOLE 1	-.0013	-.0004	(0.0000,	0.0000,	.1415)
6	HOLE 2	-.0006	-.0002	(-.0500,	.2900,	.1415)
7	HOLE 3	-.0006	-.0002	(.2120,	-.0950,	.1415)
8	HOLE 4	-.0017	-.0005	(-.0313,	-.2228,	.1415)

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** MOMENTS-II **   SAMPLE CASE 3

P R O P E R T I E S   O F   W E D G E S

NO.	IDENTIFICATION	VOLUME	MASS	C.G.(W.R.T. REF. AXES)		
1	NO. 5 L2	.0017	.0005	(-.1314,	.0646,	.1415)
2	NO. 6 L2	.0024	.0007	(-.1329,	-.0921,	.1415)
3	NO. 7 L2	.0019	.0006	(-.0834,	-.1664,	.1415)
4	NO. 9 L2	.0010	.0003	(.0935,	-.2378,	.1415)
5	NO. 10 L2	.0032	.0010	(.0808,	-.1326,	.1415)

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** MOMENTS-II **   SAMPLE CASE 3

P R O P E R T I E S   O F   C O N C A V E   P A R T S

NO.	IDENTIFICATION	VOLUME	MASS	C.G.(W.R.T. REF. AXES)		
1	NO. 1 L2	.0027	.0008	(.2387,	.0349,	.1415)
2	NO. 4 L2	.0006	.0002	(-.3330,	.2015,	.1415)

** MOME(S-II **   SAMPLE CASE 3

PROPERTIES OF ENTIRE BODY

MASS =   .0120

NET VOLUME =   .0388

POSITIVE VOLUME =   .0431

NEGATIVE VOLUME =   -.0043

COORDINATES OF CG =(.0312,   .0962,   .1415)

COMPONENTS OF INERTIA TENSOR WITH RESPECT TO REFERENCE AXES

I(XX) =   .0010
I(XY) =   -.0000
I(YY) =   .0007
I(XZ) =   -.0001
I(YZ) =   -.0002
I(ZZ) =   .0011

COMPONENTS OF INERTIA TENSOR WITH RESPECT TO C.G. AXES

I(XX) =   .0006
I(XY) =   -.0000
I(YY) =   -.0004
I(XZ) =   -.0000
I(YZ) =   -.0000
I(ZZ) =   .0010

PRINCIPAL MOMENTS   PRINCIPAL AXES

.0006   (.9996014, -.0282303, 0.0000000)

.0004   (.0282303, .9996014, 0.0000000)

.0010   (0.0000000, 0.0000000, 1.0000000)

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APPENDIX D - Sample Case 4

This Appendix contains a listing of the input cards, followed by the output from MOMENTS-II for Sample Case 4 as described in 1.5.4.

For the data cards, the numbers in the far left column are card numbers; they are listed here for easier reference and should not be considered part of the actual data punched on the input cards which begins to the right of the heavy line.

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** MOMENTS-II ** SAMPLE CASE 4

NTYPE IS 0

NTYPE IS 1

NTYPE IS 2

NTYPE IS 3

NTYPE IS 4

** MOMENTS-II **   SAMPLE CASE 4

M O M E N T S - I I   O P T I O N S

DEFAULT DENSITY =   .1000

DEFAULT NO. OF SECTORS/PART = 24

OUTPUT TYPE = COMPLETE

NO. OF BASIC PARTS = 5

NO. OF WEDGES = 0

NO. OF CONCAVE PARTS = 0

NO. OF KNOWN PARTS = 0

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** MOMENTS-II **   SAMPLE CASE 4

I N P U T   D A T A - B A S I C   P A R T S

NO.	IDENTIFICATION	LENGTH	DENSITY	DZ(INP. SYS.)	NTYPE-SECT.	TH(1)	TH(2)
1	NTYPE IS 0	1.0000	.20000	4.0000	0	24	

	R1(J) =	1.000	1.000	1.000	1.000	2.000	2.000
	R1(J) =	2.000	2.000	3.000	3.000	3.000	3.000
	R2(J) =	1.000	1.000	1.000	1.000	2.000	2.000
	R2(J) =	2.000	2.000	3.000	3.000	3.000	3.000

INPUTS RELATIVE TO AXES (1.0000, 0.0000, 0.0000), (0.0000, 1.0000, 0.0000), (0.0000, 0.0000, 1.0000)
INPUTS RELATIVE TO AXES WITH ORIGIN AT (0.0000, 0.0000, 0.0000)

| 2 | NTYPE IS 1 | 1.0000 | .10000 | 1.0000 | 1 | 4 | 45.000 135.000 |

| | R1(J) = | 1.000 |
| | R2(J) = | 2.000 |

INPUTS RELATIVE TO AXES (1.0000, 0.0000, 0.0000), (0.0000, 1.0000, 0.0000), (0.0000, 0.0000, 1.0000)
INPUTS RELATIVE TO AXES WITH ORIGIN AT (0.0000, 0.0000, 0.0000)

| 3 | NTYPE IS 2 | 1.0000 | .10000 | 1.0000 | 2 | 3 | |

| | R1(J) = | 1.000 | 0.000 | 1.000 |
| | R2(J) = | 2.000 | 0.000 | 2.000 |

TH(J) = 45.000 135.000 360.000
INPUTS RELATIVE TO AXES (1.0000, 0.0000, 0.0000), (0.0000, 1.0000, 0.0000), (0.0000, 0.0000, 1.0000)
INPUTS RELATIVE TO AXES WITH ORIGIN AT (0.0000, 0.0000, 0.0000)

| 4 | NTYPE IS 3 | 1.0000 | .10000 | 3.0000 | 3 | 2 | |

	R1(J) =	1.000	1.000	1.000	1.000	2.000	2.000
	R1(J) =	2.000	2.000	3.000	3.000	3.000	3.000
	R2(J) =	1.000	1.000	1.000	1.000	2.000	2.000
	R2(J) =	2.000	2.000	3.000	3.000	3.000	3.000
	RH(J) =	.1000	.1000	.1000	.1000	.1000	.1000
	RH(J) =	.1000	.2000	.2000	.2000	.2000	.2000

INPUTS RELATIVE TO AXES (1.0000, 0.0000, 0.0000), (0.0000, 1.0000, 0.0000), (0.0000, 0.0000, 1.0000)
INPUTS RELATIVE TO AXES WITH ORIGIN AT (0.0000, 0.0000, 0.0000)

I N P U T   D A T A - B A S I C   P A R T S

NO.	IDENTIFICATION	LENGTH	DENSITY	DZ(INP. SYS.)	NTYPE-SECT.		TH(1)	TH(2)
5	NTYPE IS 4	1.0000	.10000	0.0000	4	24		
	RI(J) = 1.000	1.000	1.000	1.000	1.000	2.000	2.000	2.000
	RI(J) = 2.000	2.000	3.000	3.000	3.000	3.000	3.000	3.000
	R2(J) = 1.000	1.000	1.000	1.000	1.000	2.000	2.000	2.000
	R2(J) = 2.000	2.000	3.000	3.000	3.000	3.000	3.000	3.000
	TH(J) = 10.000	20.000	30.000	40.000	50.000	60.000	70.000	80.000	95.000	110.000	125.000	140.000
	TH(J) = 155.000	170.000	185.000	200.000	220.000	240.000	260.000	280.000	300.000	320.000	340.000	360.000
	RH(J) = .1000	.1000	.1000	.1000	.1000	.1000	.1000	.1000	.1000	.1000
	RH(J) = .1000	.1000	.1000	.2000	.2000	.2000	.2000	.2000	.2000

INPUTS RELATIVE TO AXES (1.0000, 0.0000, 0.0000), (0.0000, 1.0000, 0.0000), (0.0000, 0.0000, 1.0000)
INPUTS RELATIVE TO AXES WITH ORIGIN AT (0.0000, 0.0000, 0.0000)

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P R O P E   I E S   O F   B A S I C   P A R T S

NO.	IDENTIFICATION	VOLUME	MASS	C.G.(W.R.T. REF. AXES)		
1	NTYPE IS	14.6608	2.9322	(.2363,	-.8867,	4.5000)
2	NTYPE IS 1	1.8326	.1833	(.0000,	.9646,	1.6071)
3	NTYPE IS 2	5.4978	.5498	(-.0000,	-.3215,	1.6071)
4	NTYPE IS 3	14.6608	2.4086	(.4674,	-1.1002,	3.5000)
5	NTYPE IS 4	17.4533	3.0020	(.0991,	-1.0550,	.5020)

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** MOMENTS-II **   SAMPLE CASE 4

PROPERTIES OF ENTIRE BODY

MASS = 9.0757

NET VOLUME = 54.1052

POSITIVE VOLUME = 54.1052

NEGATIVE VOLUME = 0.0000

COORDINATES OF CG =(.2329, -.9274, 2.6779)

COMPONENTS OF INERTIA TENSOR WITH RESPECT TO REFERENCE AXES

I(XX) = 110.5389
I(XY) = 3.4047
I(YY) = 106.8011
I(XZ) = -7.2054
I(YZ) = 22.5585
I(ZZ) = 32.7912

COMPONENTS OF INERTIA TENSOR WITH RESPECT TO C.G. AXES

I(XX) = 37.6503
I(XY) = 1.4448
I(YY) = 41.2265
I(XZ) = -1.5463
I(YZ) = .0187
I(ZZ) = 24.4930

PRINCIPAL MOMENTS   PRINCIPAL AXES

37.3061   (.9324702, -.3431085, -.1130307)

41.7520   (.3420055, .9392309, -.0296222)

24.3117   (.1163256, -.0110353, .9931498)

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